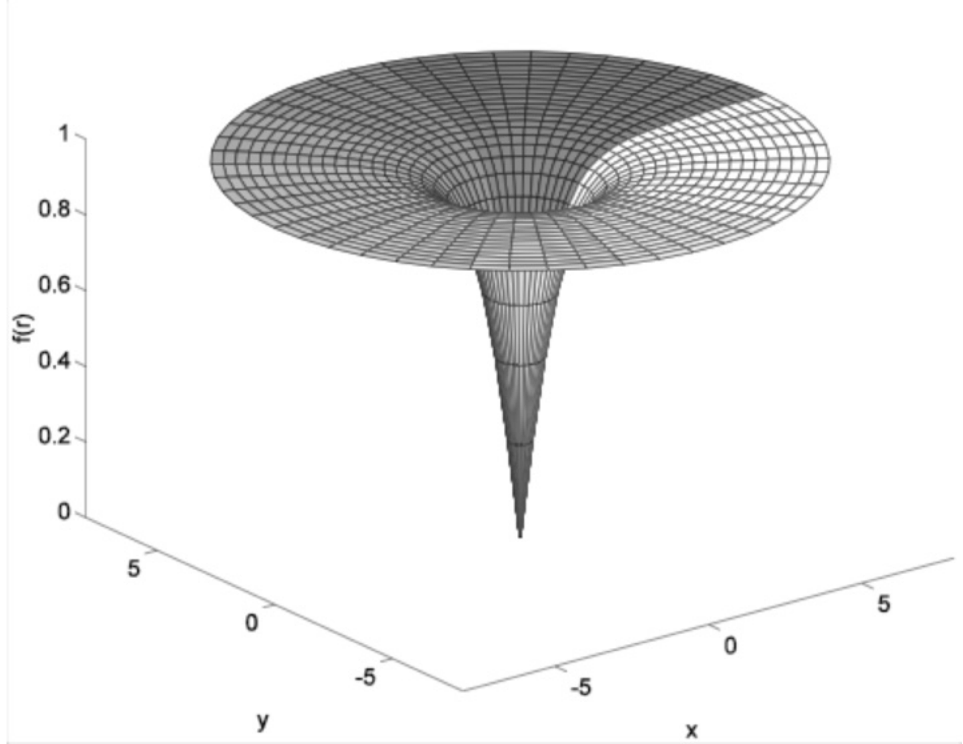
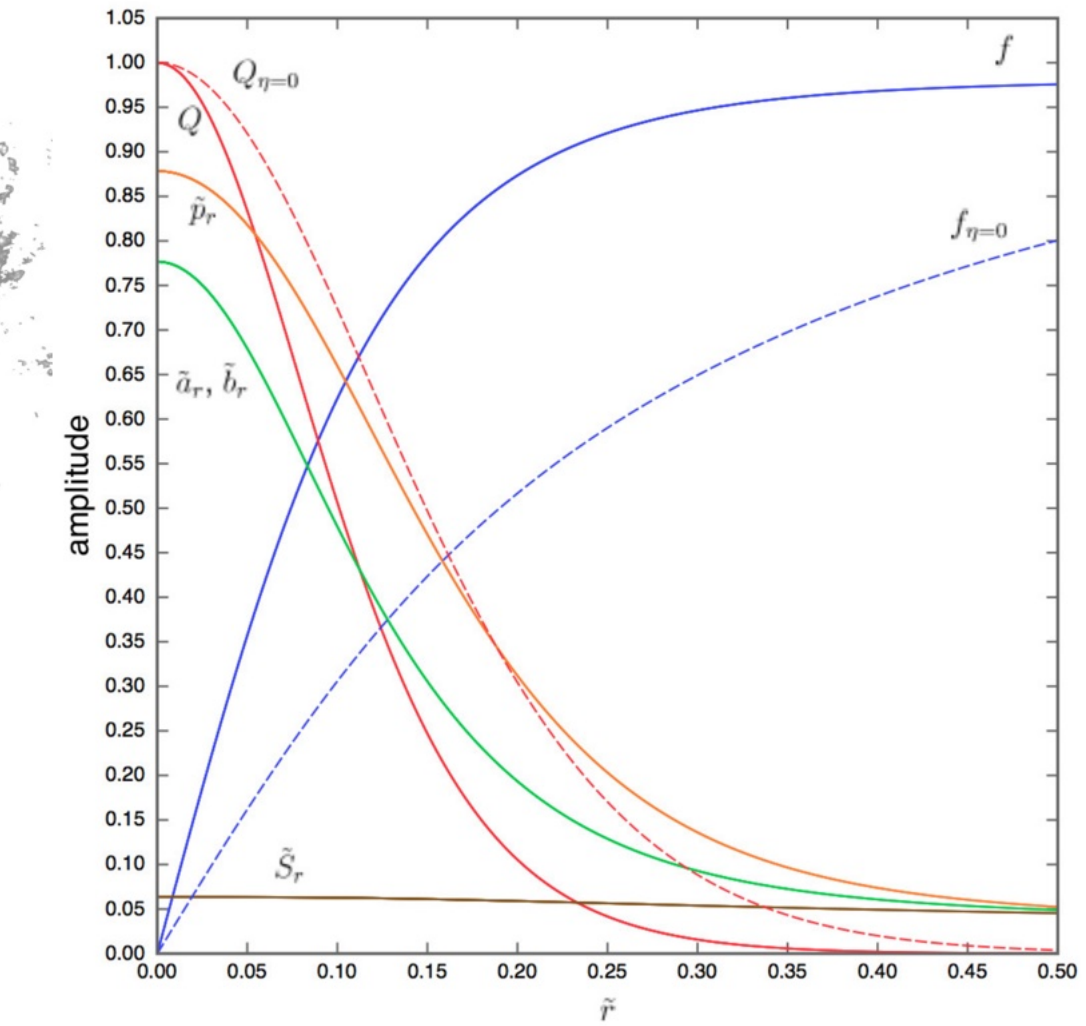
PATRICK PETER

GReCO

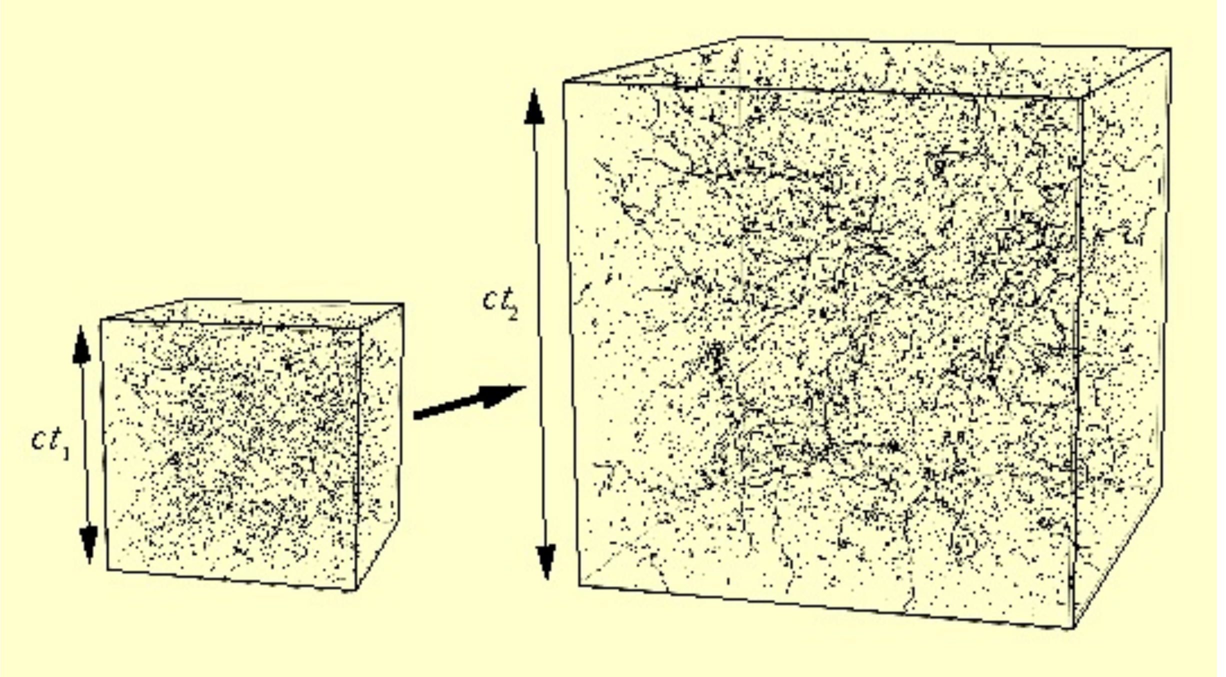
Institut d'Astrophysique de Paris





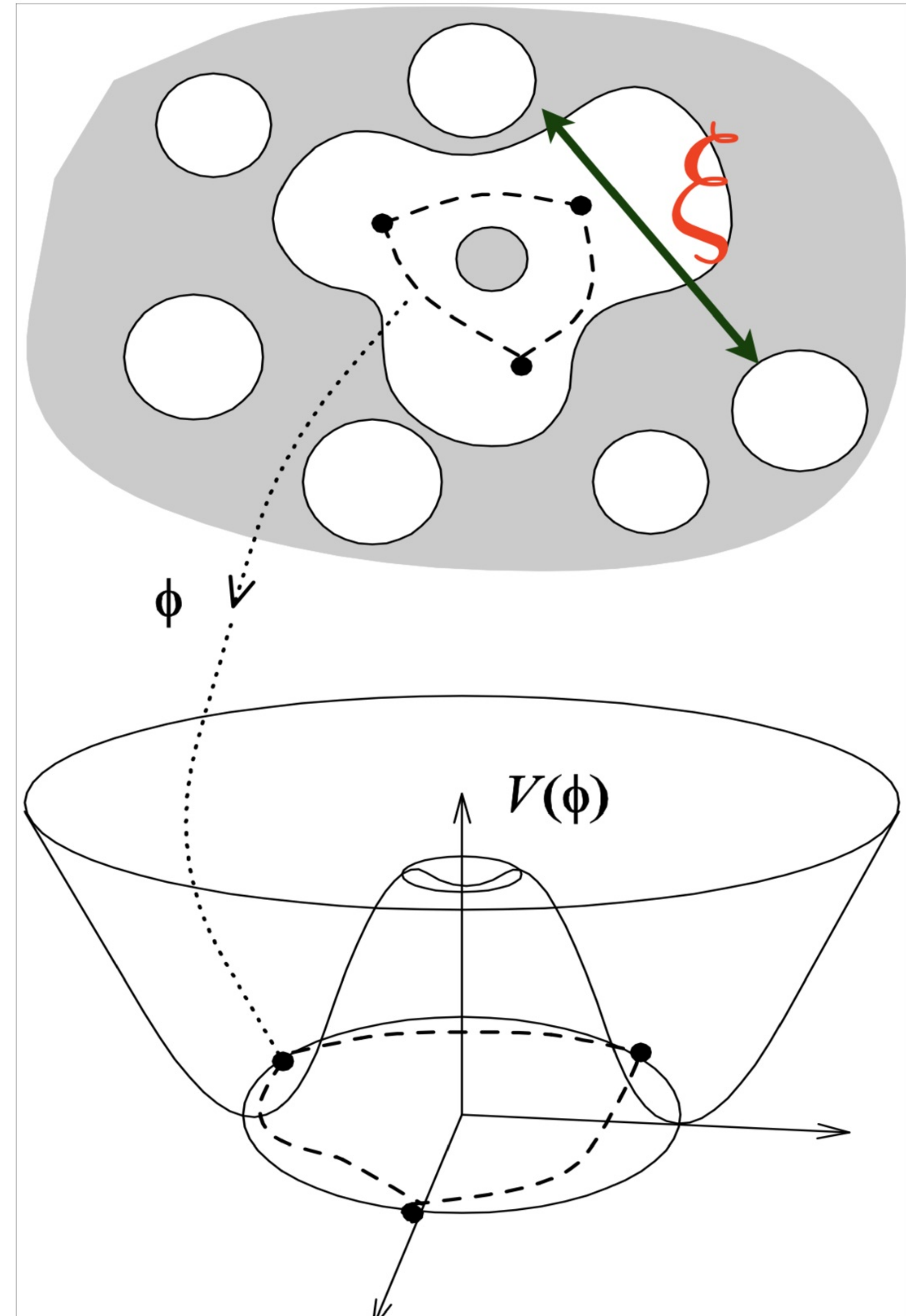
The CVOS model for superconducting cosmic string networks

Chacun a son défaut, où toujours il revient
J. de la Fontaine, Fables, 1668



Montpellier – 27/11/2023

Phase transition



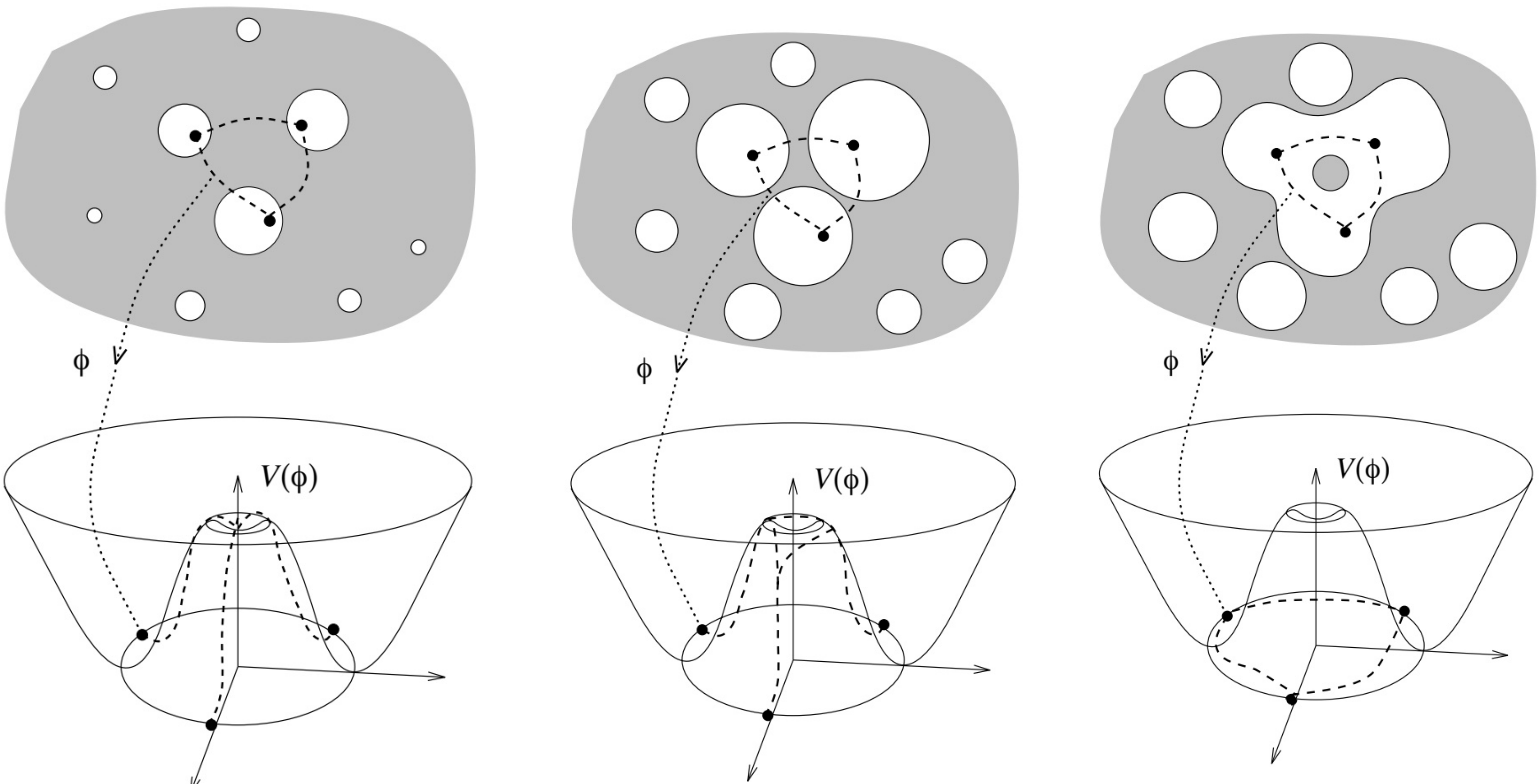
Correlation length

$$\langle 0|\phi(\vec{x})\phi(\vec{x}+\vec{r})|0\rangle \propto \mathrm{e}^{-|\vec{r}|/\xi}$$

Model: cosmic string

T.W.B. Kibble, *J. Phys.* **A9**, 1387 (1976)
M.B. Hindmarsh and T.W.B. Kibble, *Rep. Prog. Phys.* **58**, 477 (1995)

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}D_\mu\Phi\left(D^\mu\Phi\right)^* - V(\Phi)$$



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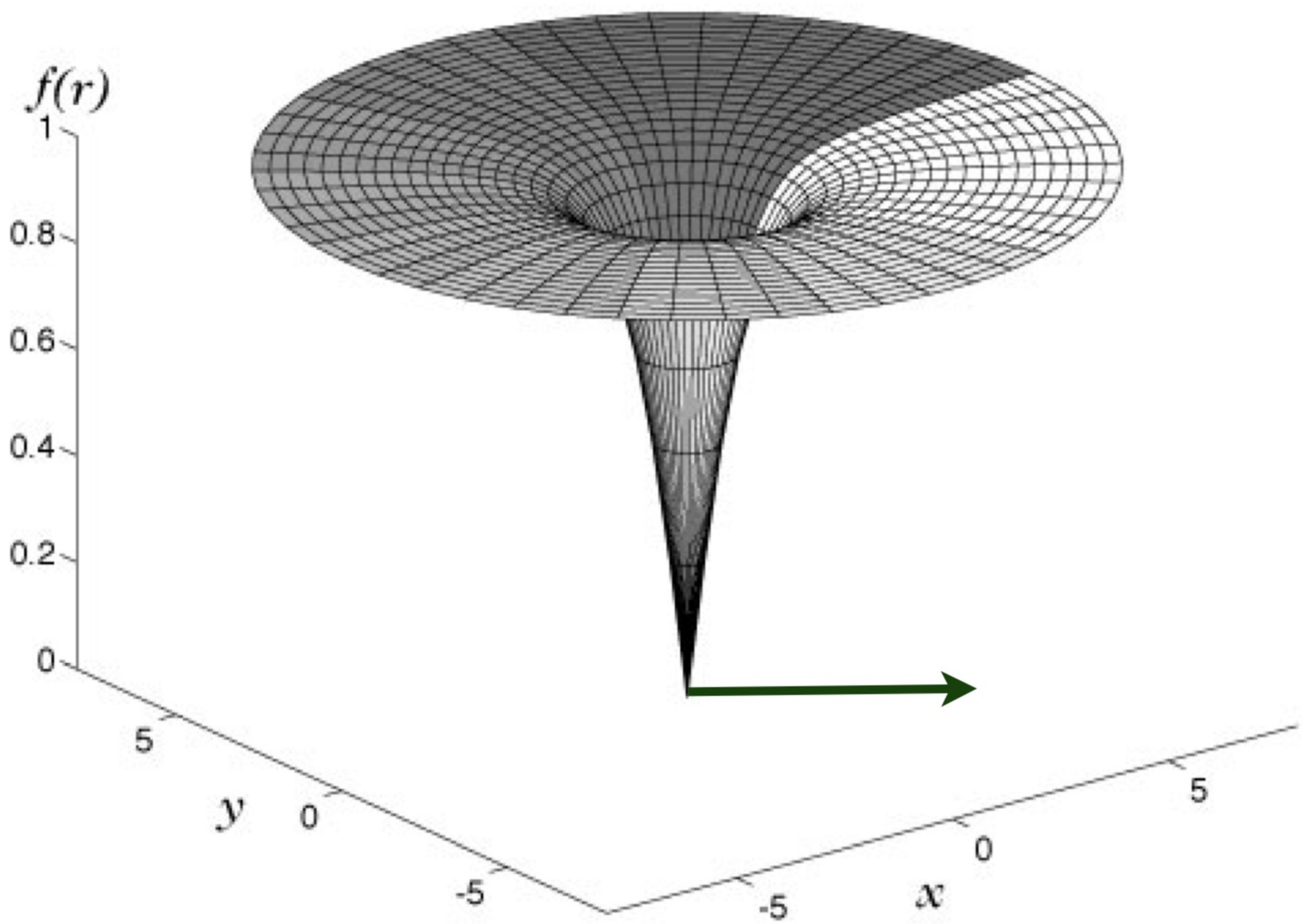
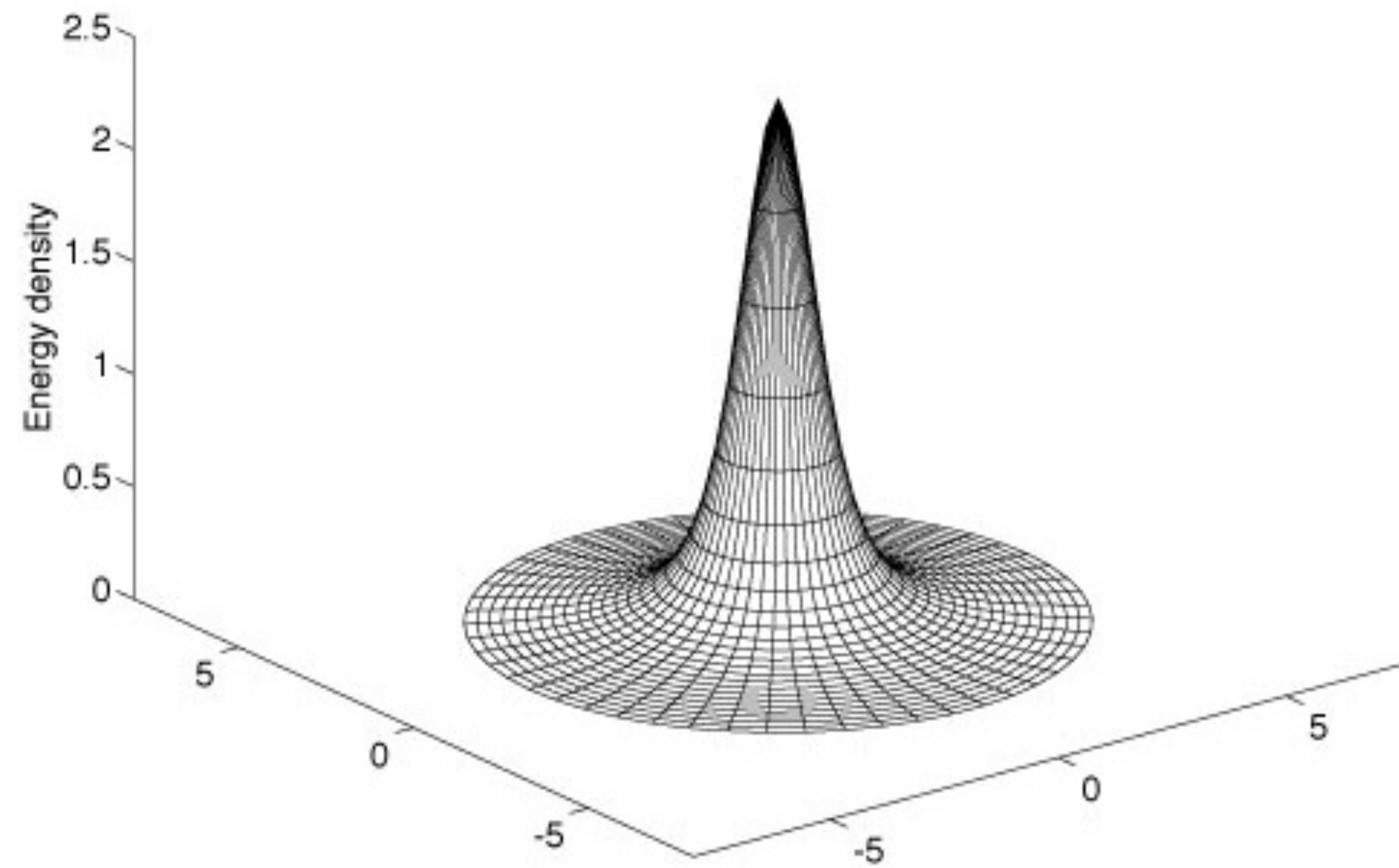
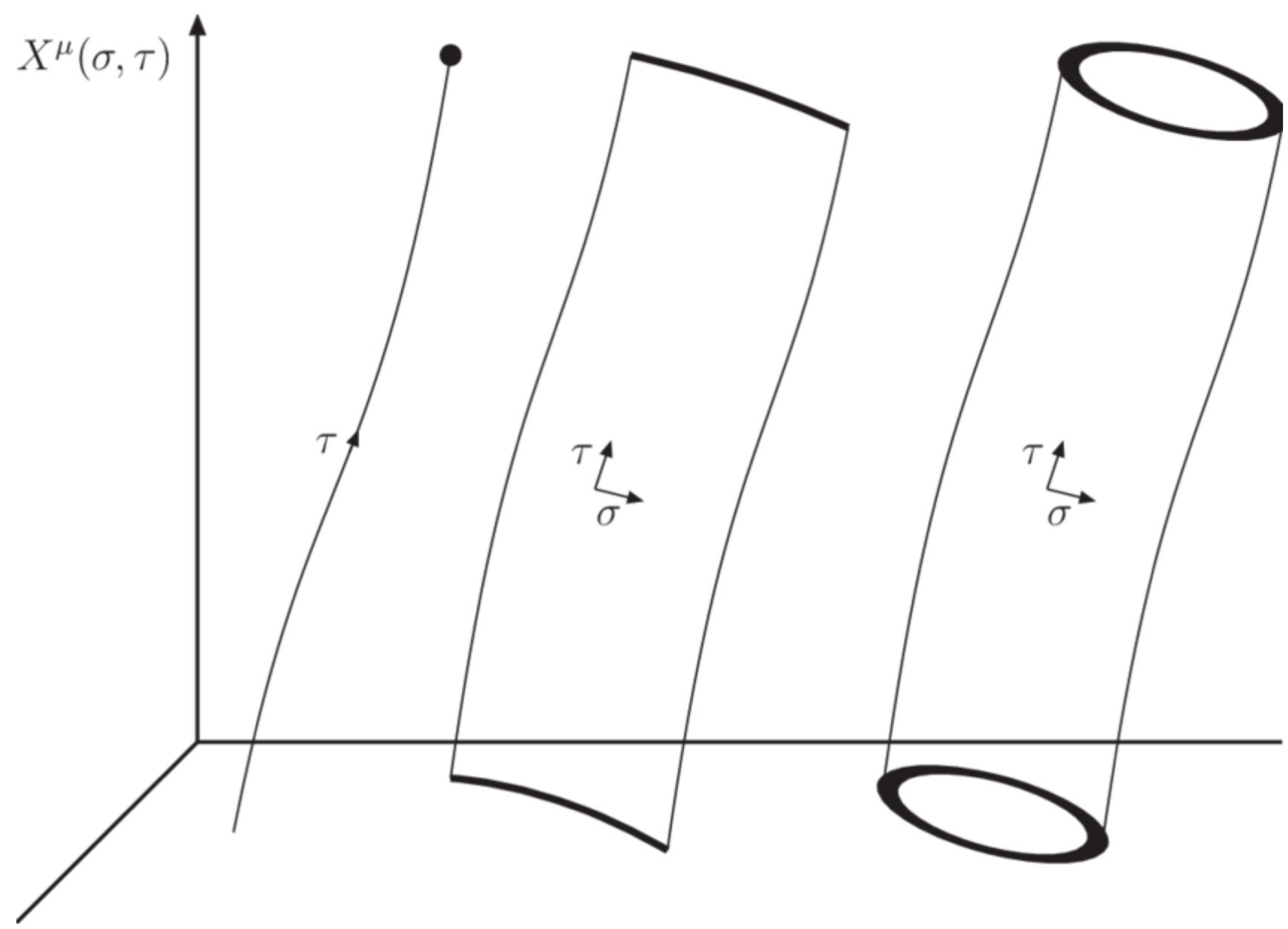


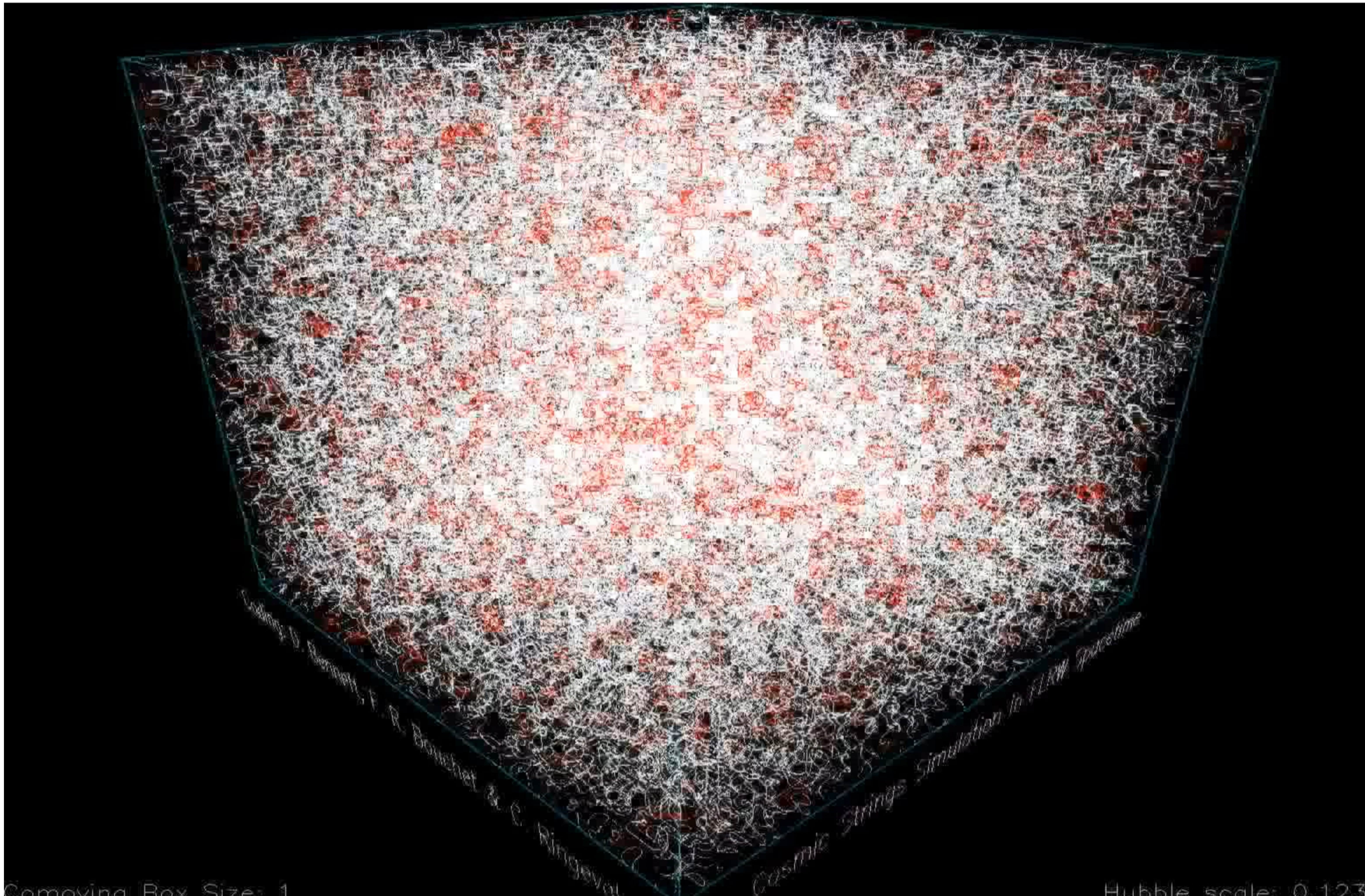


$\Phi \to f(r)\mathrm{e}^{in\theta}$ Nielsen-Olesen ansatz

Nambu-Goto strings

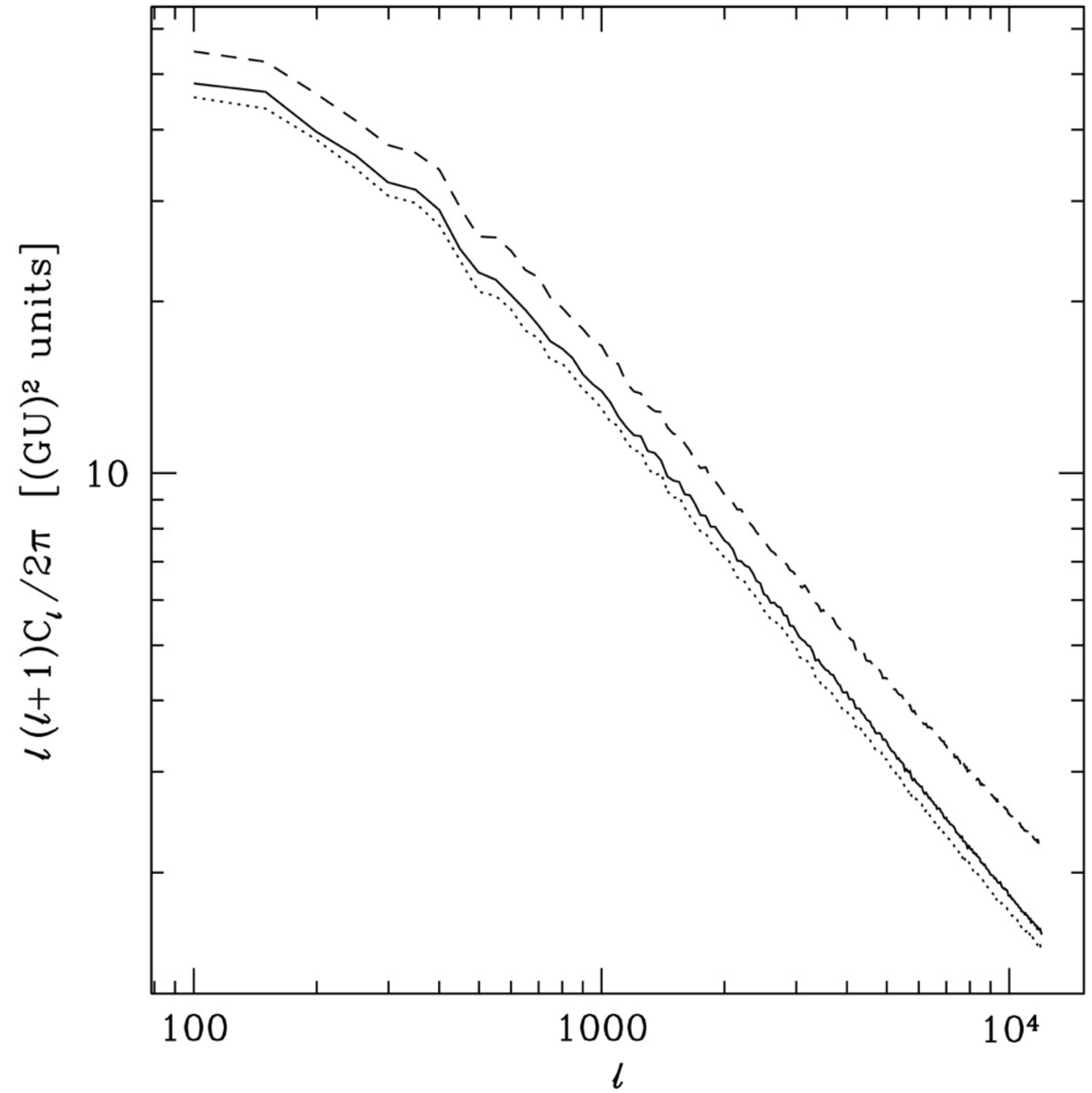
$$\mathcal{S} = \int \sqrt{-\gamma}\,\mathrm{d}\sigma\mathrm{d}\tau \quad \Longrightarrow \quad \ddot{\boldsymbol{x}} = \boldsymbol{x}''$$

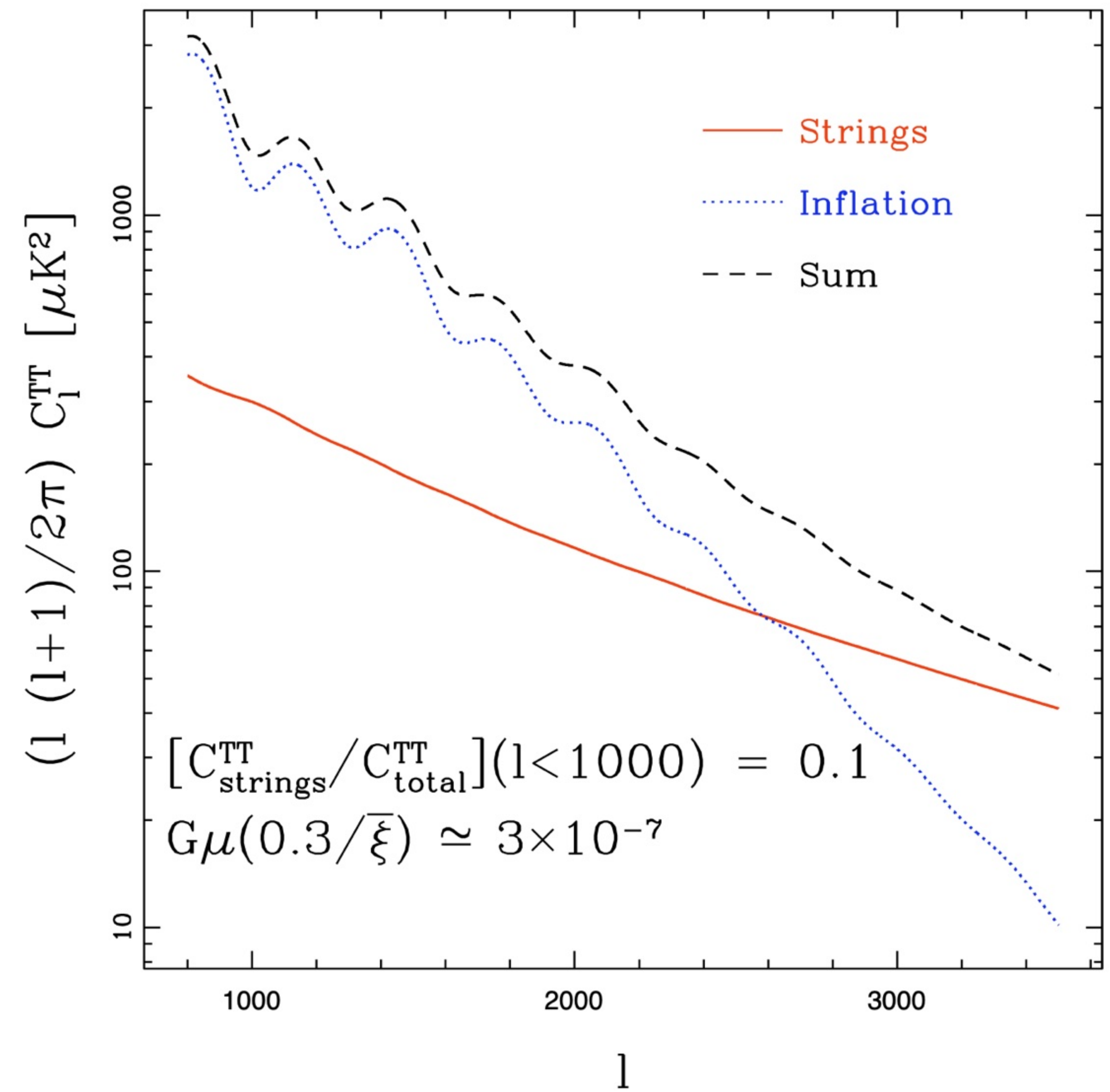




https://commons.wikimedia.org/wiki/File:Cosmic_strings_evolution_during_the_expansion_of_the_Universe.webm

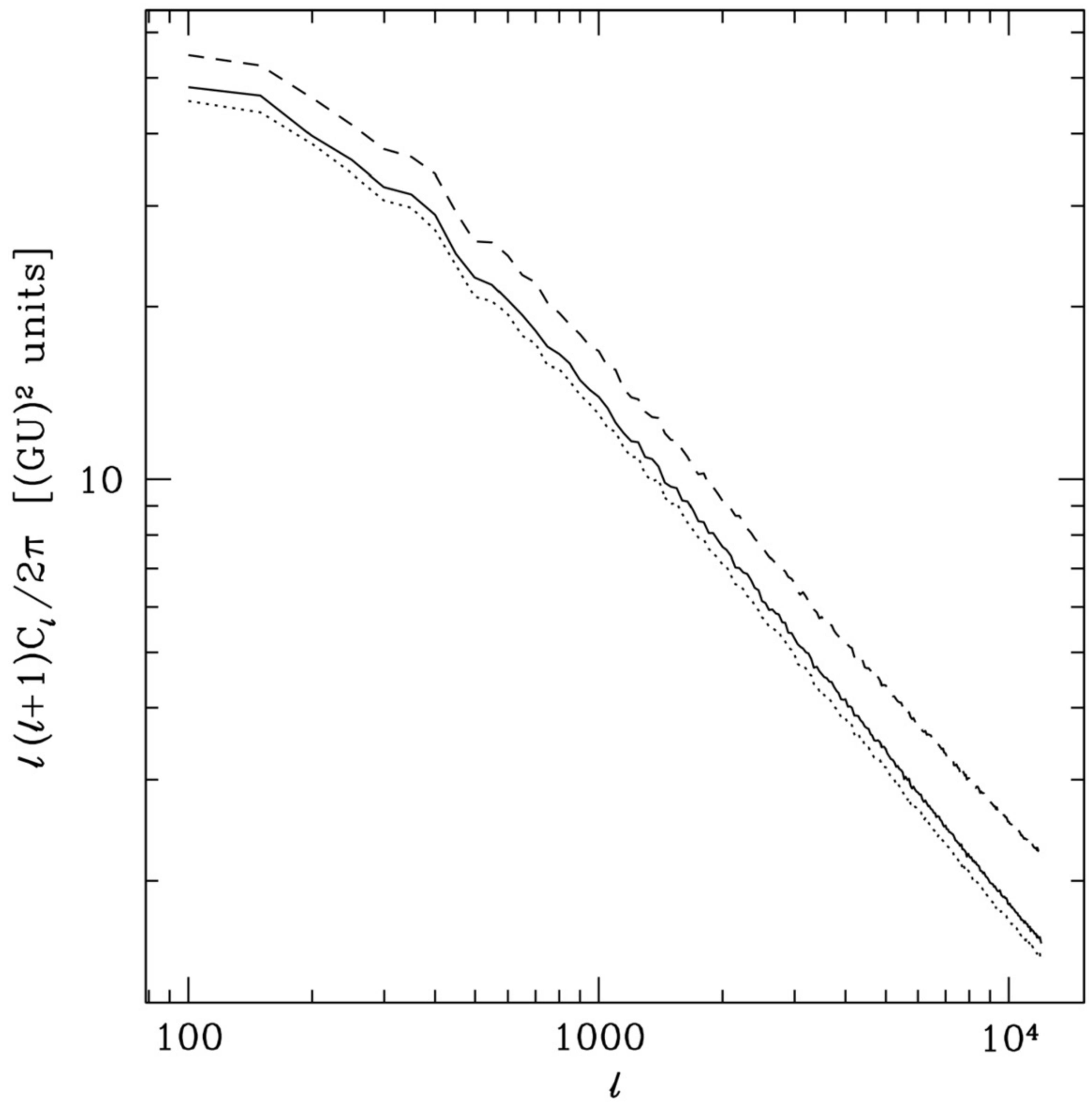
Fraisse, Ringeval, Spergel & Bouchet (2008)

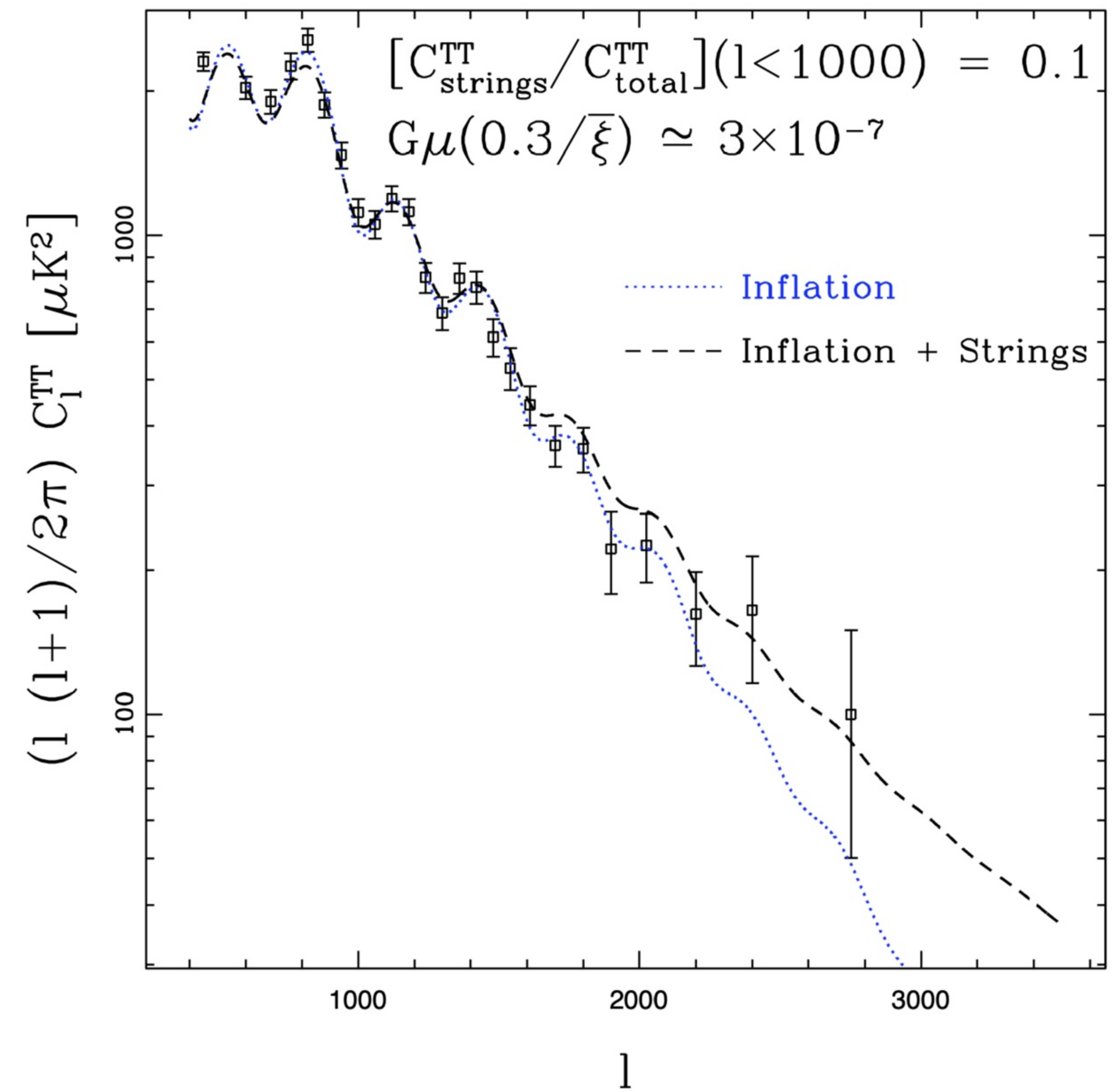




Pogosian et al. (2008)

Fraisse, Ringeval, Spergel & Bouchet (2008)

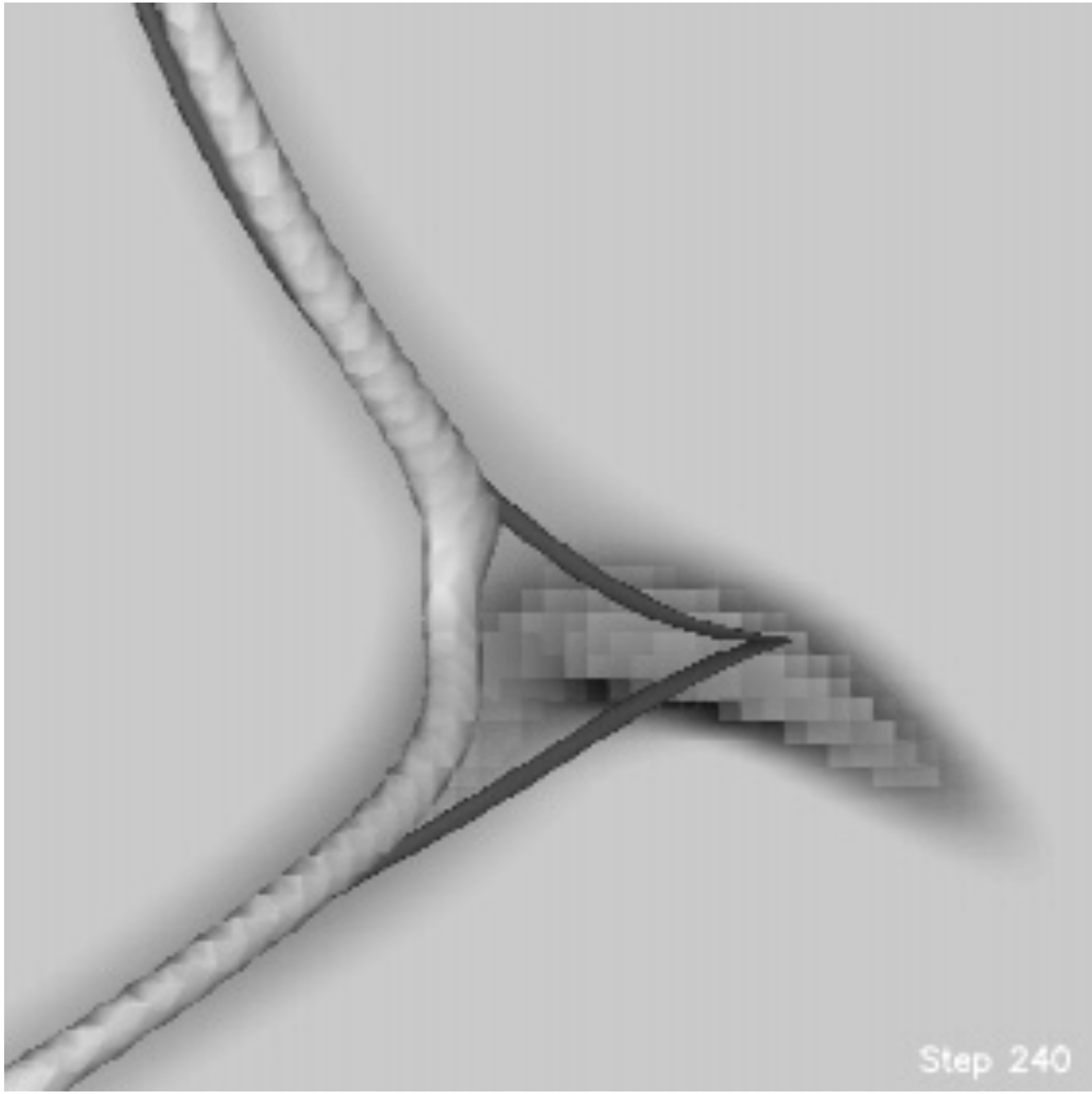




Pogosian et al. (2008)

GW stochastic background

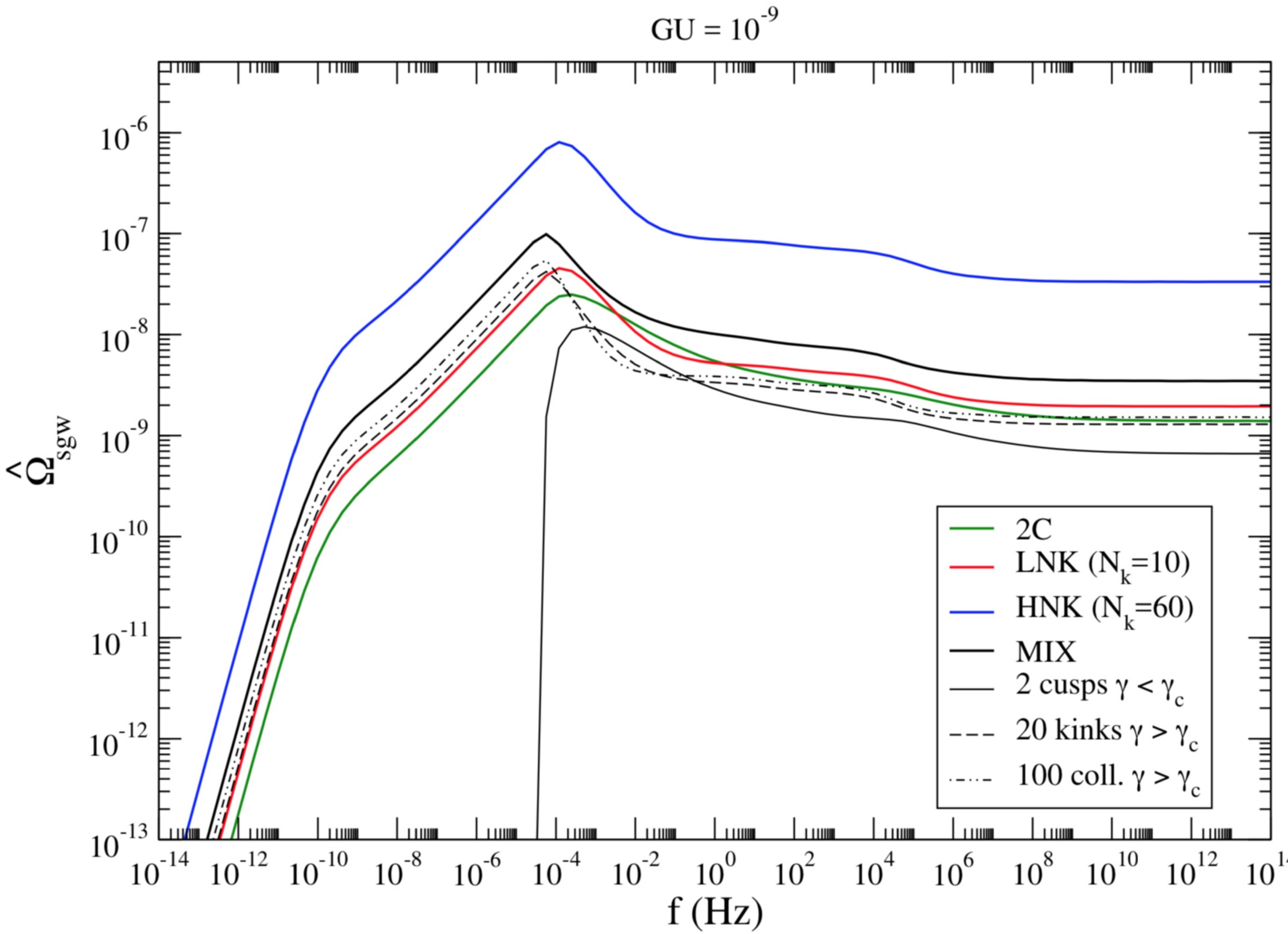
Mostly based on existence of cusps…

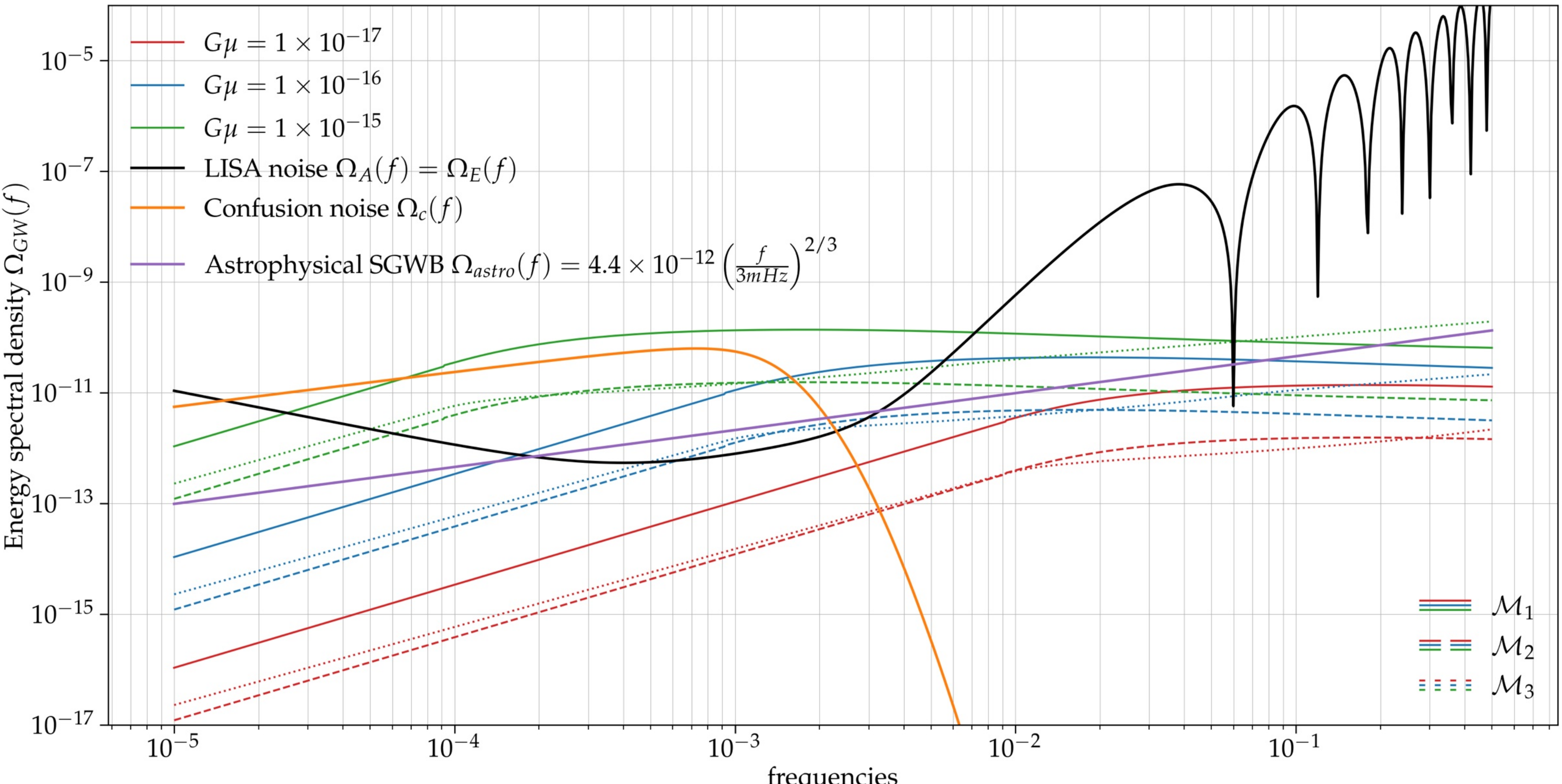


K. D. Olum & J. J. Blanco-Pillado, *Phys. Rev.* **D60,** 023503 (1999)

GW stochastic background

C. Ringeval & T. Suyama, *JCAP* **17,** 12 (2017)
J. J. Blanco-Pillado & K. D. Olum, *Phys. Rev.* **D96**, 104046 (2017)
J. J. Blanco-Pillado K. D. Olum & X. Siemens, *Phys. Lett.* **B778,** 392 (2018)





$G\mu = 1 \times 10^{-17}$
$G\mu = 1 \times 10^{-16}$
$G\mu = 1 \times 10^{-15}$
LISA noise $\Omega_A(f) = \Omega_E(f)$
Confusion noise $\Omega_c(f)$
Astrophysical SGWB $\Omega_{astro}(f) = 4.4 \times 10^{-12} \left(\frac{f}{3mHz}\right)^{2/3}$
$\mathcal{M}_1$
$\mathcal{M}_2$
$\mathcal{M}_3$
Energy spectral density $\Omega_{GW}(f)$
frequencies
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}

Model: Witten superconducting cosmic string

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}D_\mu\Phi\left(D^\mu\Phi\right)^* + \frac{1}{2}D_\mu\Sigma\left(D^\mu\Sigma\right)^* - V(\Phi,\Sigma)$$

Field strength tensors

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Covariant derivatives

$$D_\mu\Phi = \left(\partial_\mu - \mathsf{i}\, e_\phi B_\mu\right)\Phi$$

$$D_\mu\Sigma = \left(\partial_\mu - \mathsf{i}\, e_\sigma A_\mu\right)\Sigma$$

Potential

$$V(\Phi,\Sigma) = \frac{\lambda_\phi}{4}(|\Phi|^2-\eta^2)^2 + f(|\Phi|^2-\eta^2)|\Sigma|^2 + \frac{\lambda_\sigma}{4}|\Sigma|^4 + \frac{m_\sigma^2}{2}|\Sigma|^2$$

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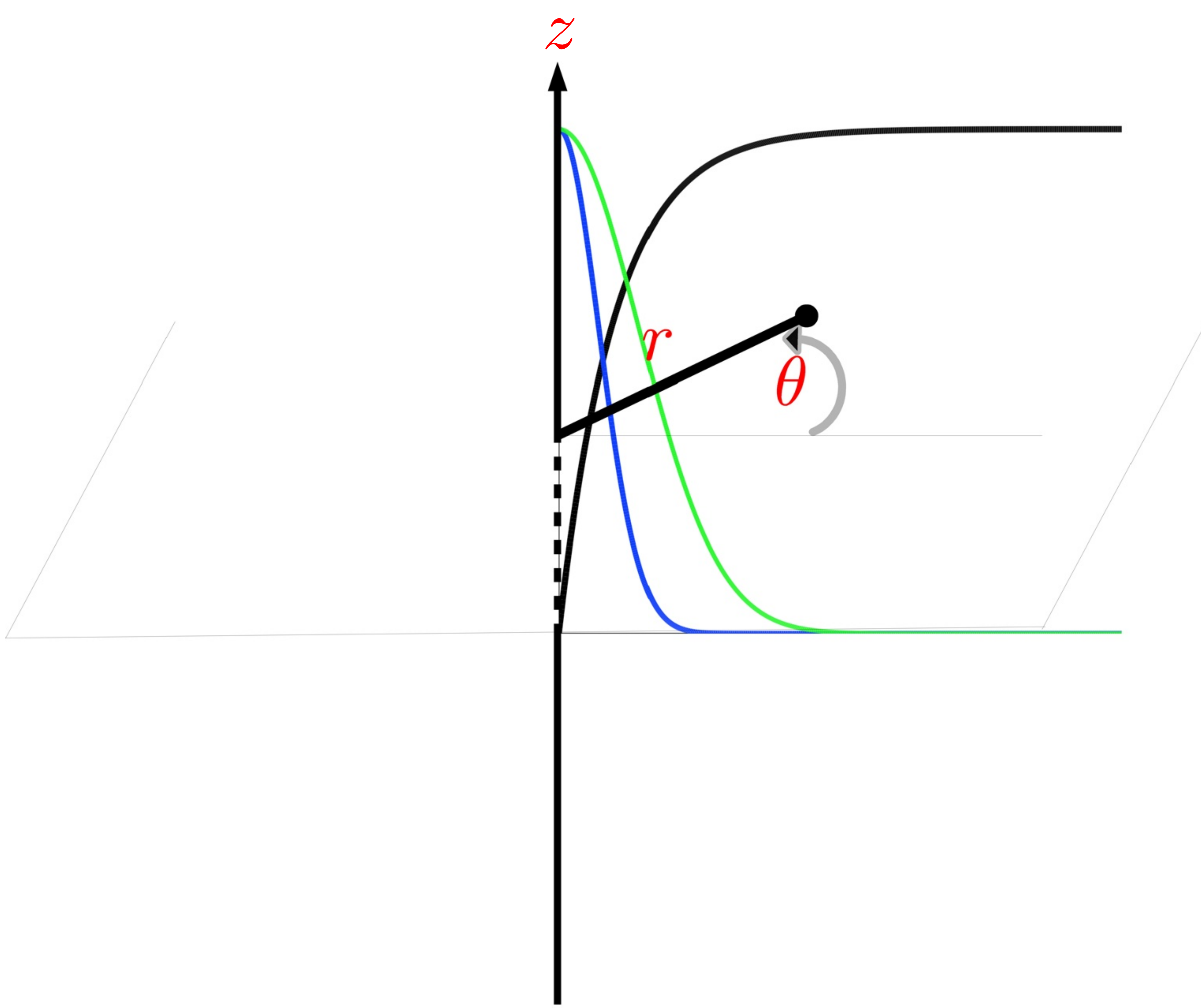
Cylindrical coordinates (t, r, θ, z)



Ansatz

$$B_\mu \mathrm{d}x^\mu = \frac{1}{e_\phi}\left[n - P(r)\right]\mathrm{d}\theta$$

$$\Phi(r,\theta) = \eta\phi(r)\mathrm{e}^{\mathrm{i}n\theta}$$

$$A_\mu \mathrm{d}x^\mu = A_z(r)\mathrm{d}z + A_t(r)\mathrm{d}t$$

$$\Sigma(t,r,z) = \eta\sigma(r)\mathrm{e}^{\mathrm{i}(\omega t - kz)}$$

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Cylindrical coordinates (t, r, θ, z)

Background and condensate

$$\Phi(r,\theta) = \eta\phi(r)\mathrm{e}^{\mathrm{i}n\theta}$$

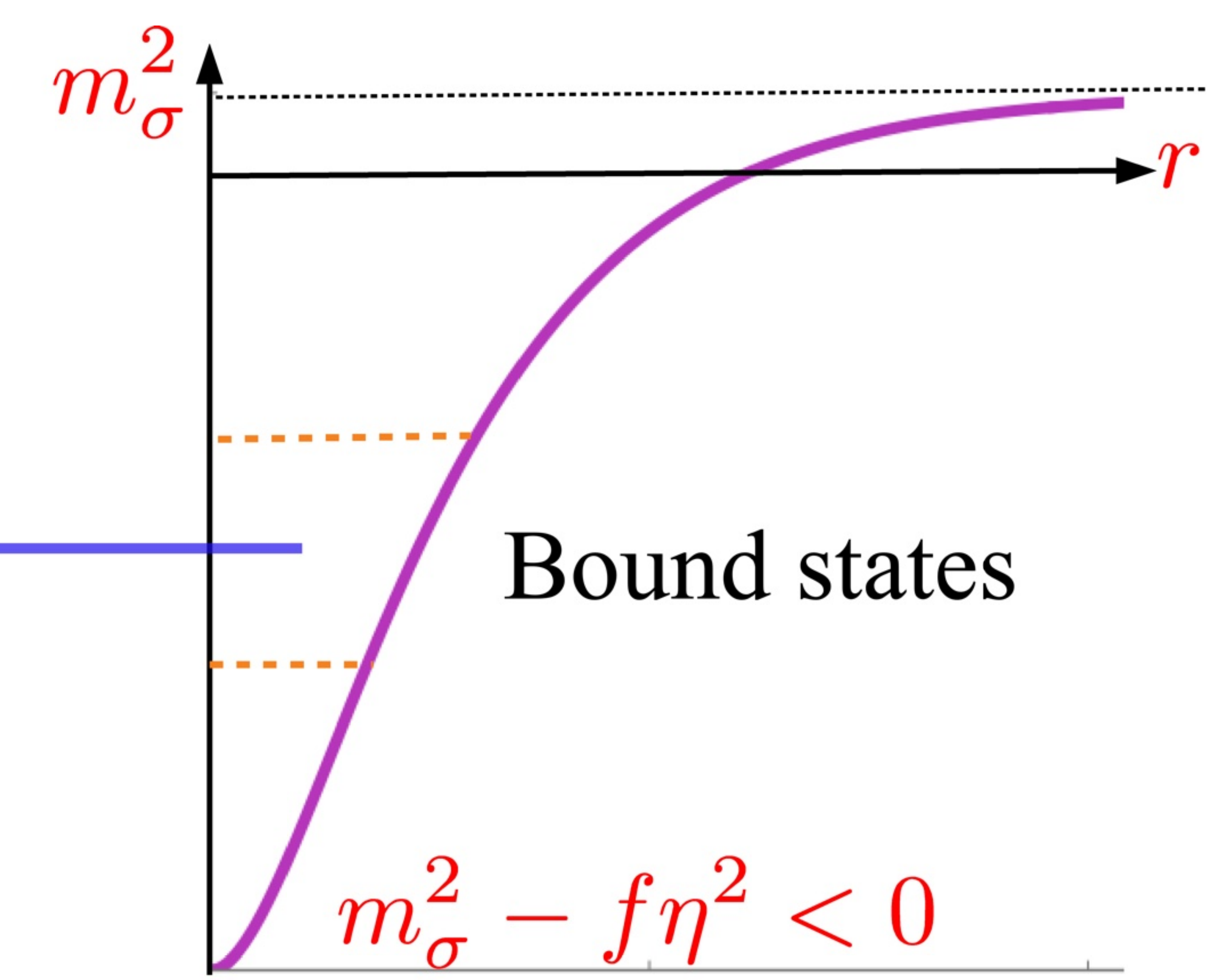
$$\Sigma(t,r,z) = \eta\sigma(r)\mathrm{e}^{\mathrm{i}Et}$$

$\Longrightarrow$

2D Schrödinger equation

$$\left[-\Delta + V(r)\right]\sigma = E^2\sigma$$

$$V(r) = f\left[\phi^2(r) - \eta^2\right] + m_\sigma^2$$



Negative energy solutions
$\Longrightarrow$ instabilities

E. Witten, *Nucl. Phys.* **B249**, 557 (1985)

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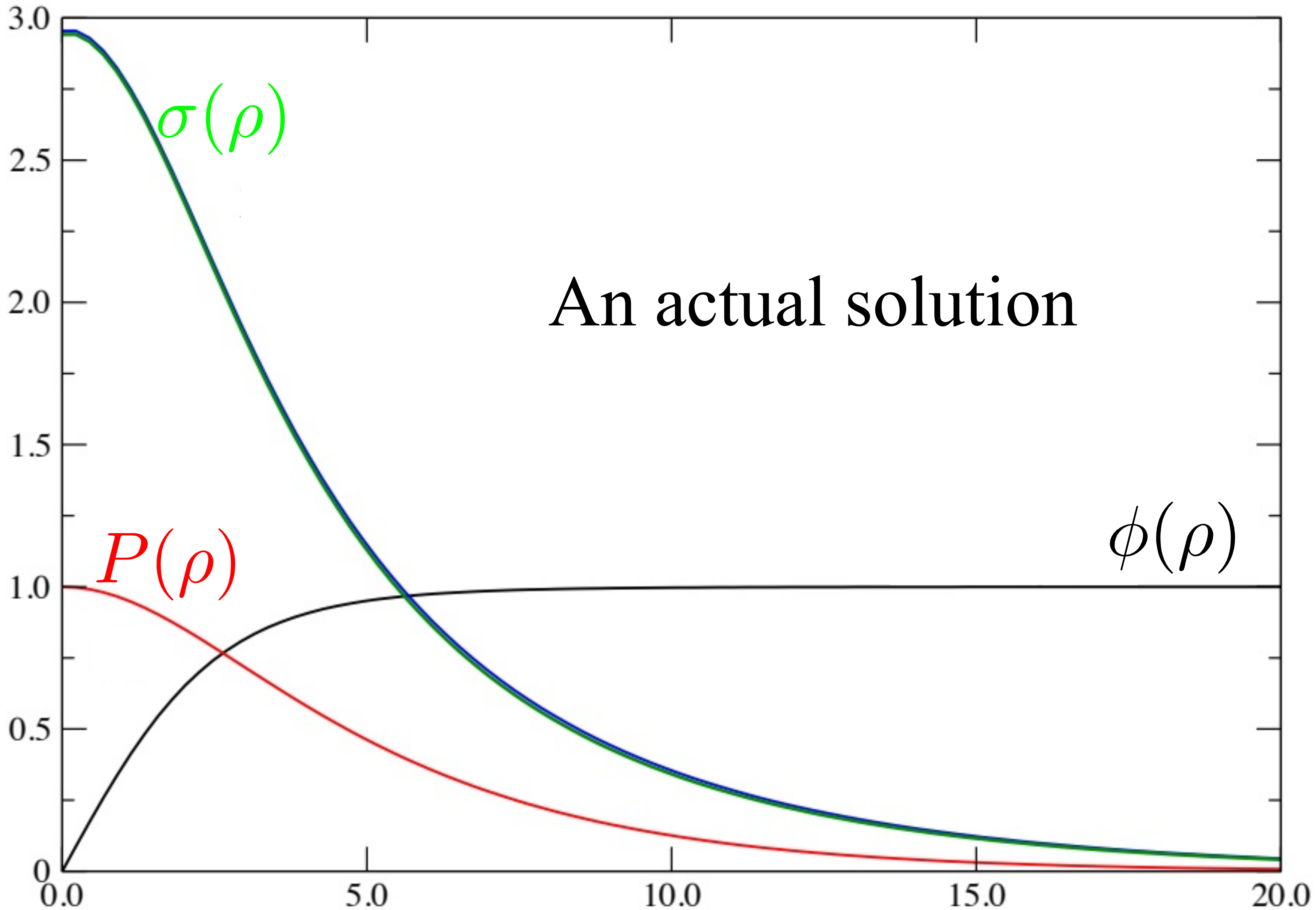

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$$\rho \equiv \sqrt{\lambda_\phi}\eta r$$

PP, *Phys. Rev.* **D45**, 1091 (1992)

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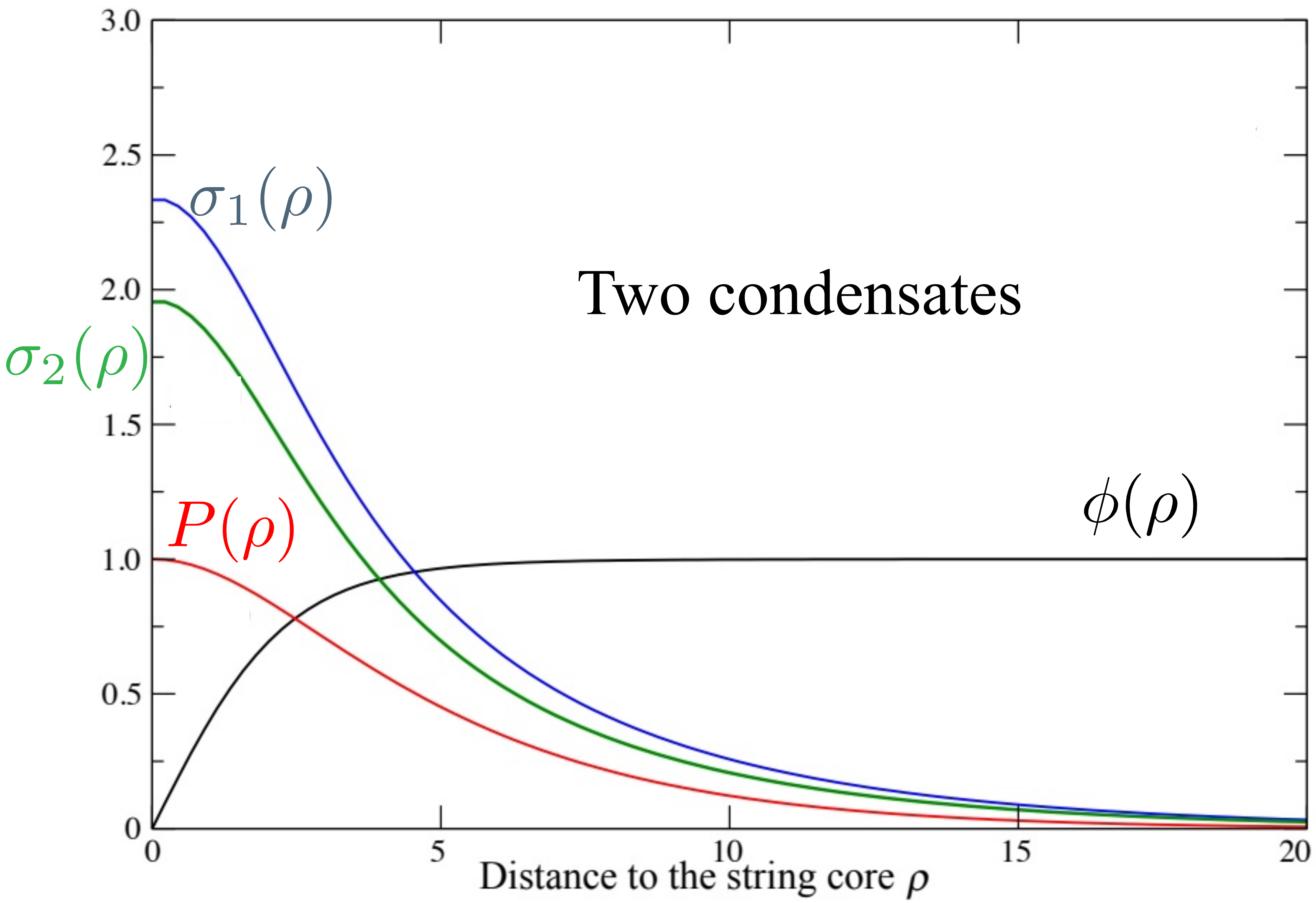
Cylindrical coordinates (t, r, θ, z)



M. Lilley, X. Martin & PP, *Phys. Rev.* **D79**, 103514 (2009)

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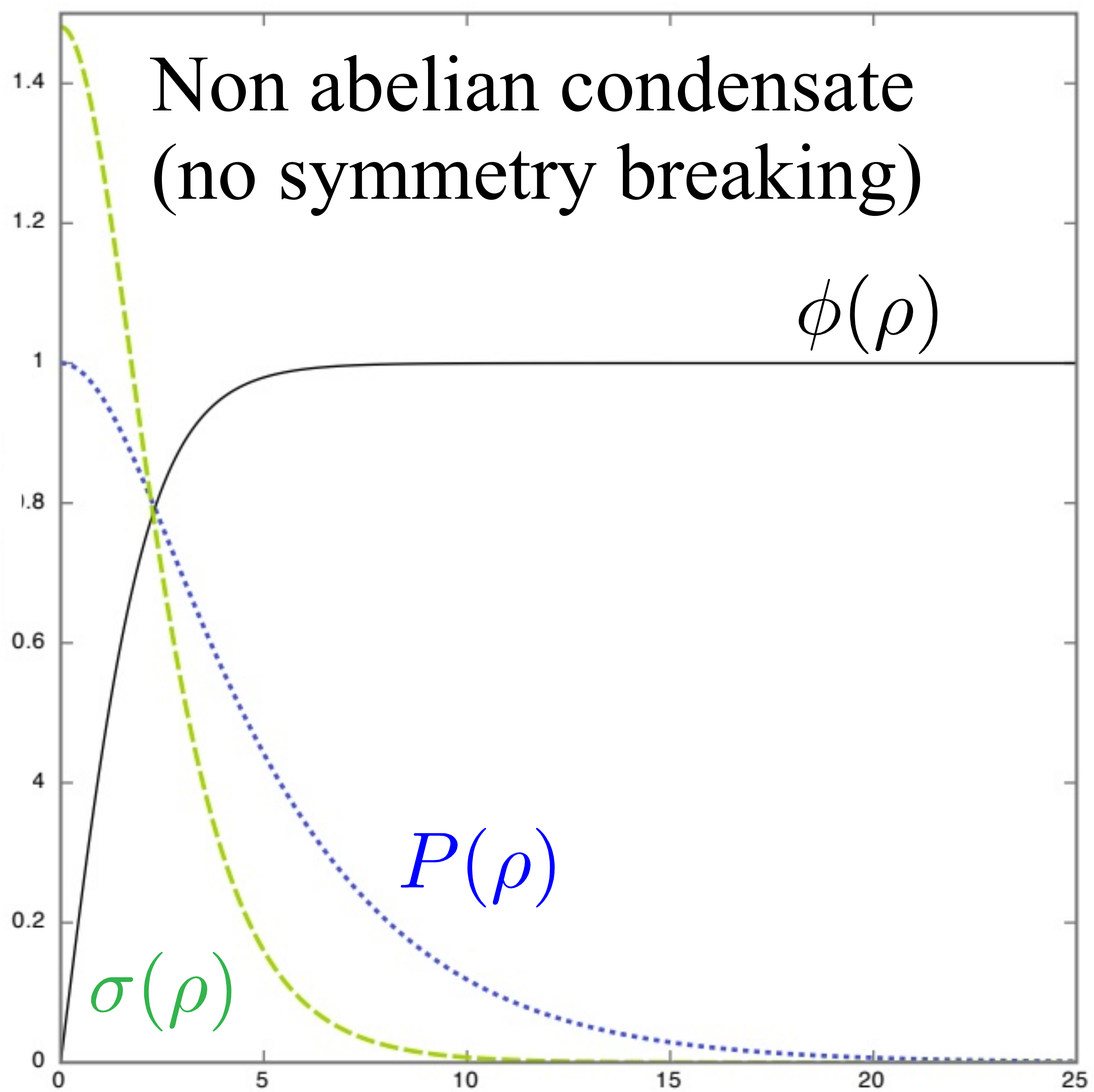

M. Lilley, F. Di Marco, J. Martin & PP, *Phys. Rev.* **D82**, 023510 (2010)

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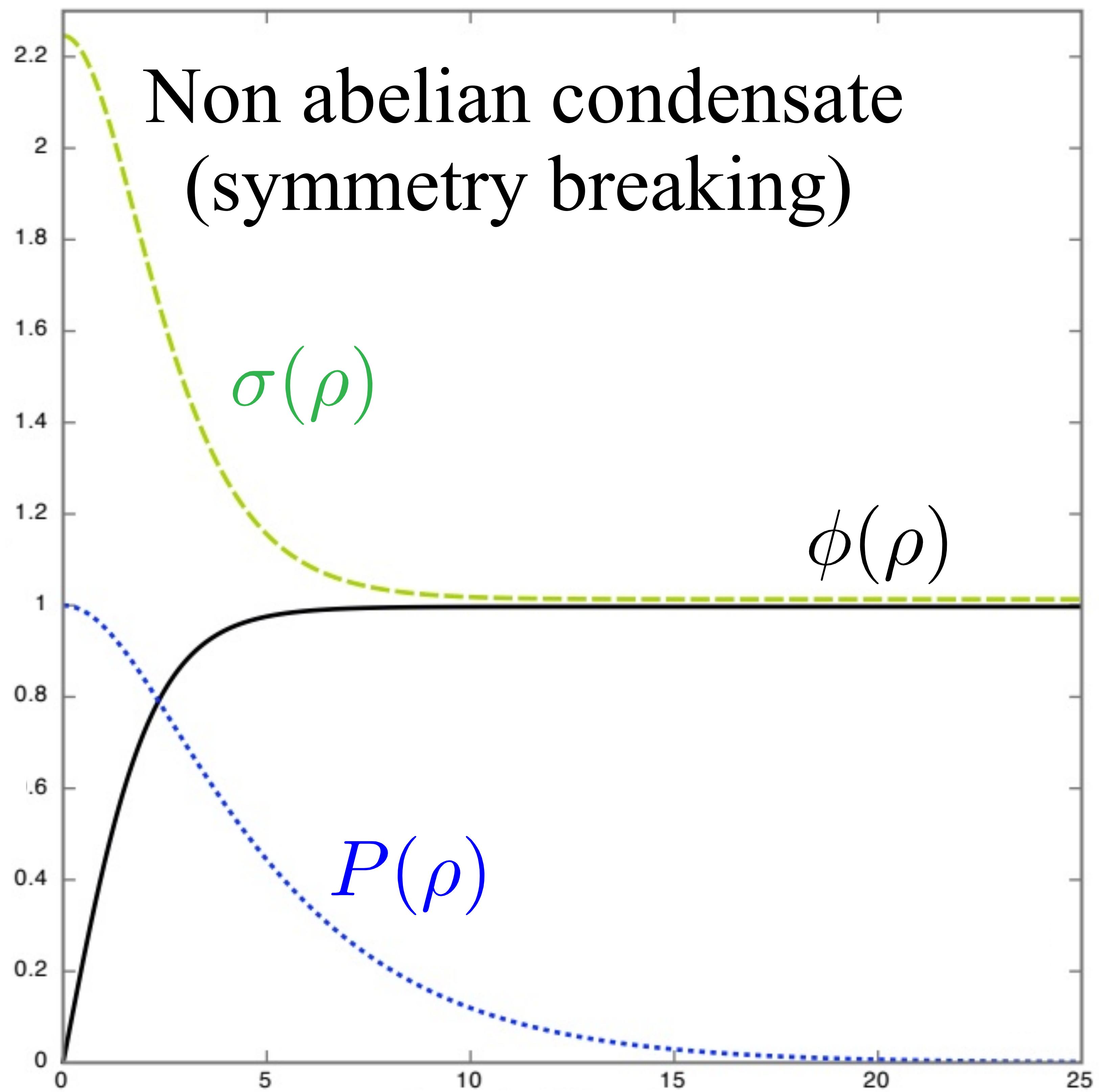

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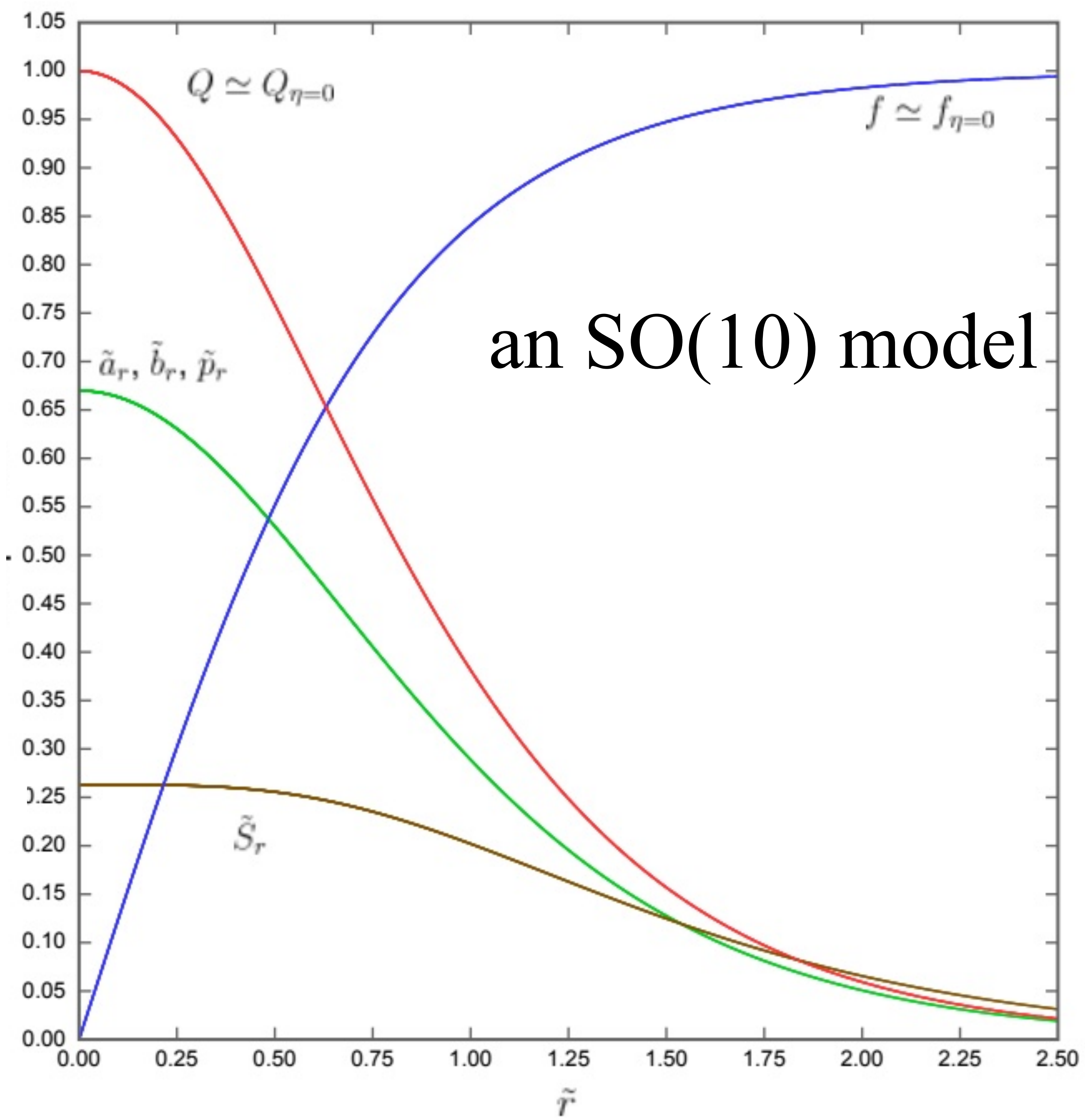
Cylindrical coordinates (t, r, θ, z)



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A.-C. Davis & S. C. Davis, *Phys. Rev.* **D55**, 1879 (1997)
E. Allys, *Phys. Rev.* **D93**, 105021 (2016)

Model: Witten superconducting cosmic string

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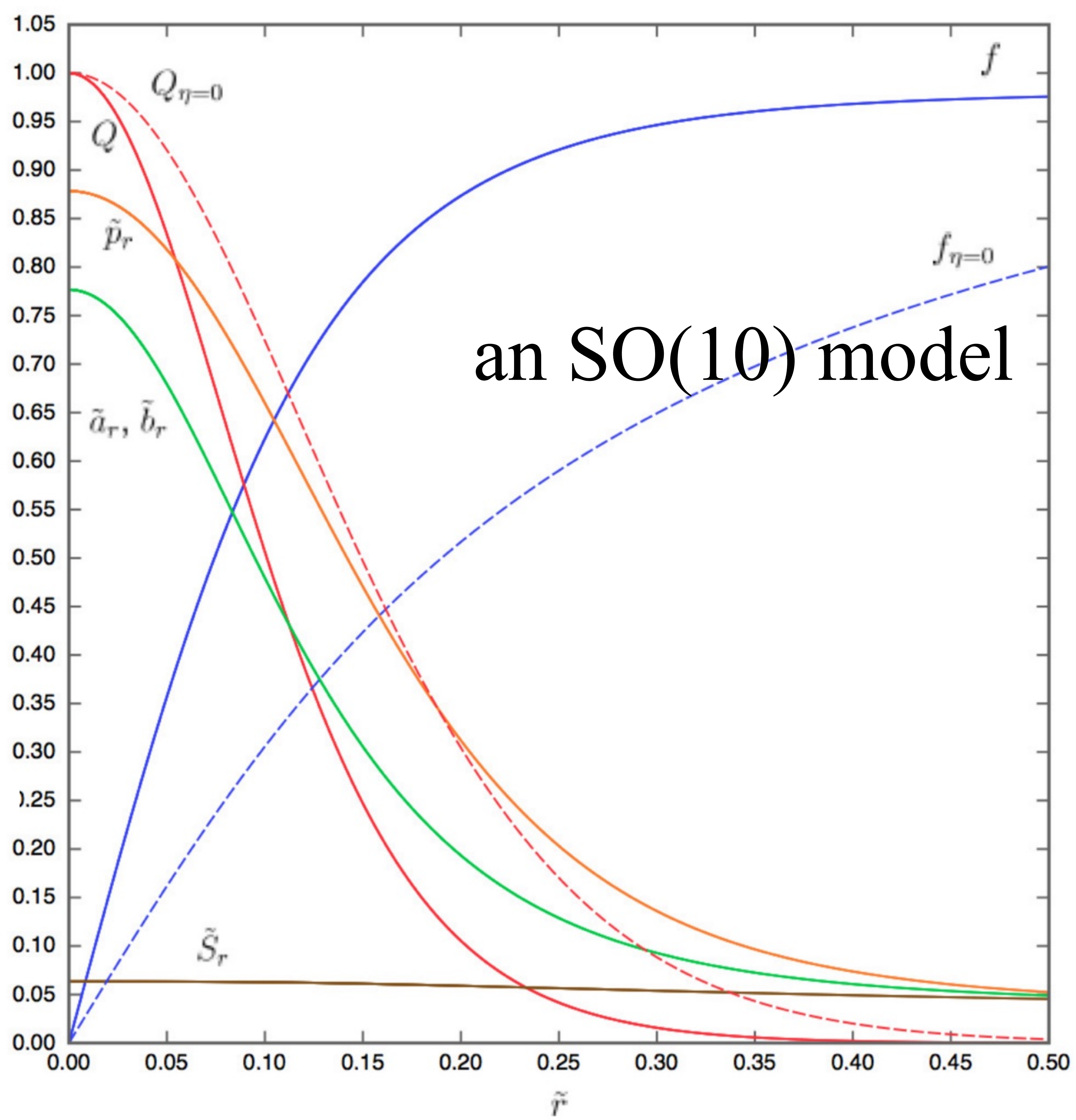
Cylindrical coordinates (t, r, θ, z)



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$$\rho \equiv \sqrt{\lambda_\phi}\eta r$$

A.-C. Davis & S. C. Davis, *Phys. Rev.* **D55**, 1879 (1997)
E. Allys, *Phys. Rev.* **D93**, 105021 (2016)

Neutral limit: (avoid complications due to long-range interaction...) PP, *Phys. Rev.* **D46**, 3335 (1992) *Phys. Rev.* **D47**, 3169 (1993)

$$e_\sigma \to 0$$

$$A_\mu \to 0$$

$$\Sigma(x^\alpha) = \sigma(x^\perp)\mathrm{e}^{\mathrm{i}\psi(\xi_a)}$$

Conserved current $\mathcal{J}_\mu = \sigma^2(r)\partial_\mu\psi$

$$\omega t_\mu - k z_\mu = \omega \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} - k \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Integrated (macroscopic) current $c_\mu \equiv \int \mathrm{d}^2x^\perp \mathcal{J}_\mu = 2\pi\partial_\mu\psi \int \sigma^2(r) r\mathrm{d}r$

Scalar current $\mu = \sqrt{|c_\mu c^\mu|}$

Scalar state parameter $\kappa = g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi \to k^2 - \omega^2$

$$\mu = 2\pi\sqrt{|\kappa|}\int \sigma^2(r) r\mathrm{d}r$$

ν ; k ; $-\omega$

Stress energy tensor $T^{\mu\nu} = -2\frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} + g^{\mu\nu}\mathcal{L}$

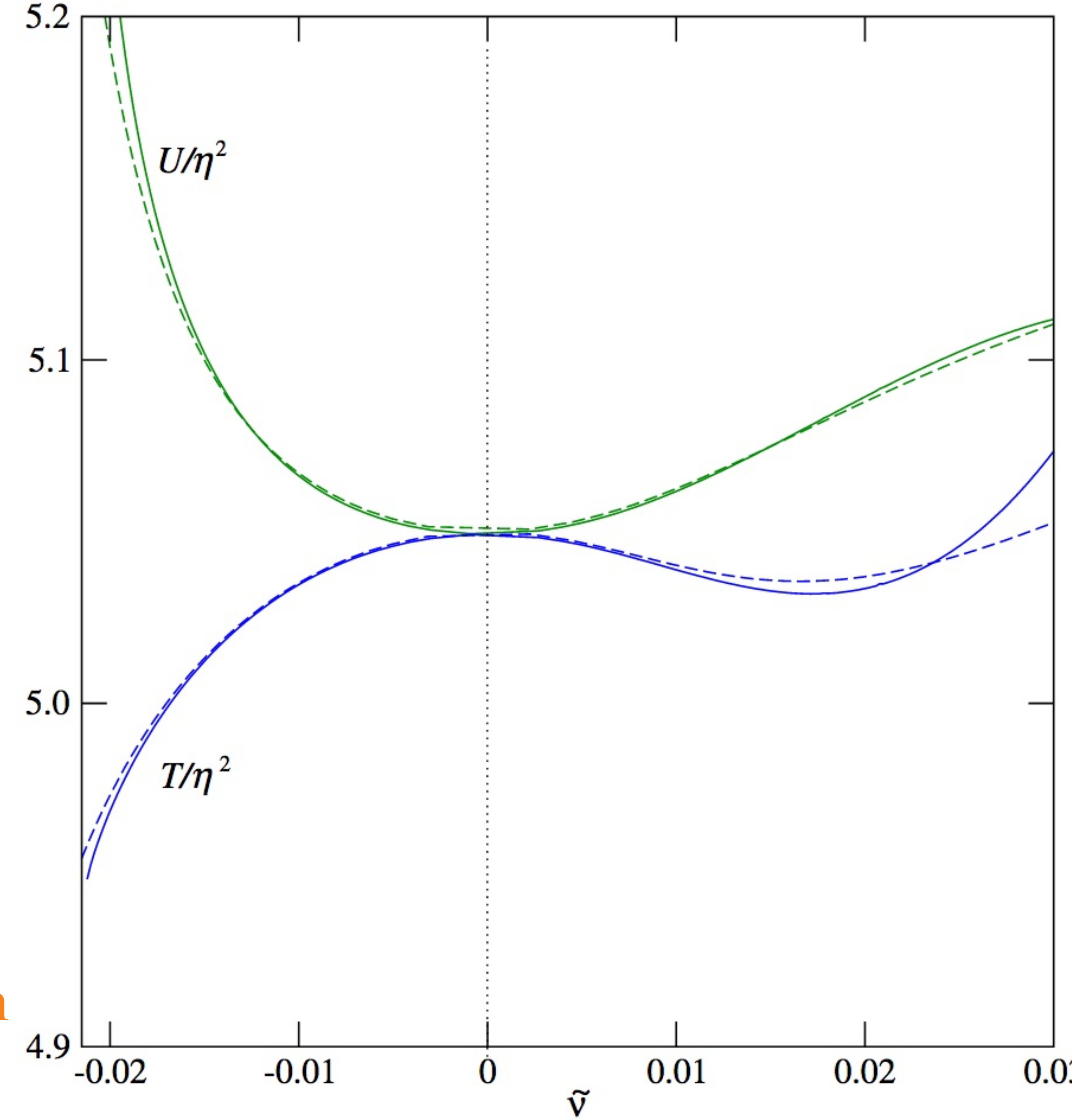
Integrated for macroscopic treatment

$$\mathcal{T}^{\mu\nu} \equiv \int T^{\mu\nu} \mathrm{d}^2 x^{\perp}$$
$$= U u^\mu u^\nu - T v^\mu v^\nu$$
$$= (U - T) u^\mu u^\nu - T \eta^{\mu\nu}$$

$U(\nu)$ $T(\nu)$

$$U - T = \nu\mu$$

Legendre transform $\Longrightarrow$ dual formalism



$$U = T + \mu\nu$$

Legendre transform $\Longrightarrow$ dual formalism

B. Carter & PP, *Phys. Rev.* **D52**, R1744 (1995)
B. Carter, PP & A. Gangui, *Phys. Rev.* **D55**, 4647 (1997)

Macroscopic formalism

State parameter $\kappa \Longrightarrow$ worldsheet lagrangian $\mathcal{L}(\kappa)$ and $\kappa = \kappa_0 \gamma^{ab} \partial_a \varphi \partial_b \varphi$

$$\nu^2 = |w|$$

$$\mathcal{S}_\mathcal{L} = -\mu_0 \int \mathrm{d}^2\xi \sqrt{-\gamma} \mathcal{L}(\kappa)$$

$$\mu^2 = |\chi|$$

Master function (dual to lagrangian) $\Lambda(\chi)$: set $\chi = \tilde{\kappa}_0 \gamma^{ab} \partial_a \psi \partial_b \psi$

$\longrightarrow$ 2 conserved (orthogonal: $\gamma_{ab} n^a z^a = 0$) currents

$$z^a = -\frac{\partial \mathcal{L}}{\partial \partial_a \varphi} \qquad n^a = -\frac{\partial \Lambda}{\partial \partial_a \psi}$$

Equivalent alternative dynamical description

$$\mathcal{S}_\mathcal{L} \Longleftrightarrow \mathcal{S}_\Lambda = -\mu_0 \int \mathrm{d}^2\xi \sqrt{-\gamma} \Lambda(\chi)$$

Current amplitudes $\mathcal{K} z^a = \kappa_0 \partial^a \varphi \quad \Longleftrightarrow \quad \tilde{\mathcal{K}} n^a = \tilde{\kappa}_0 \partial^a \psi$

$$\mathcal{K}^{-1} = -2\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\kappa} \quad \Longleftrightarrow \quad \tilde{\mathcal{K}}^{-1} = -2\frac{\mathrm{d}\Lambda}{\mathrm{d}\chi}$$

Equivalent formulations

$$\tilde{\mathcal{K}} = -\mathcal{K}^{-1} \quad \Longrightarrow \quad \kappa = \mathcal{K}^2 \chi$$

Legendre transformation $\Lambda = \mathcal{L} + \mathcal{K}\chi$

$$U = T + \mu\nu$$

$$\mu = \frac{\mathrm{d}U}{\mathrm{d}\nu}$$

$$\nu = -\frac{\mathrm{d}T}{\mathrm{d}\mu}$$

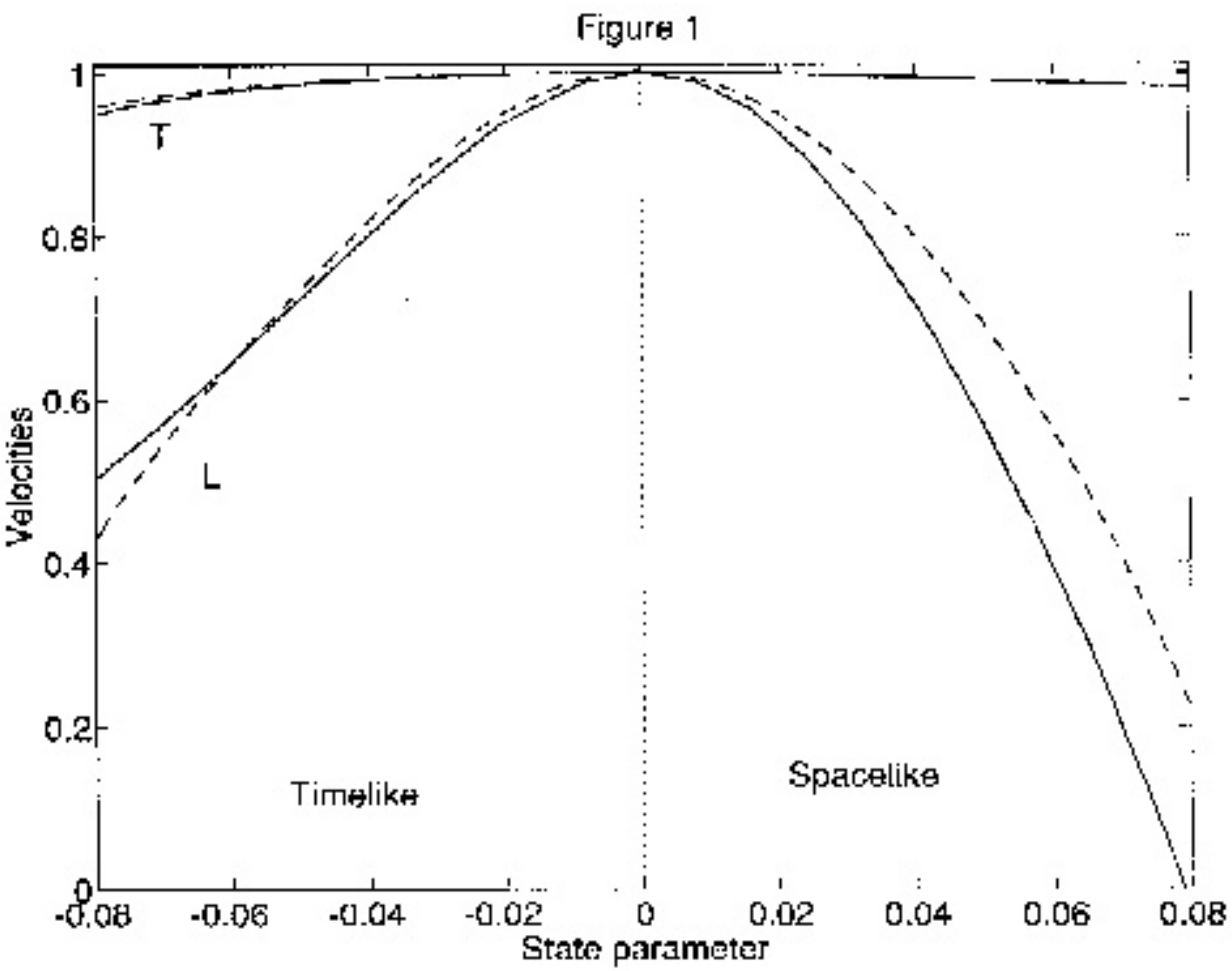
Identification for the macroscopic description

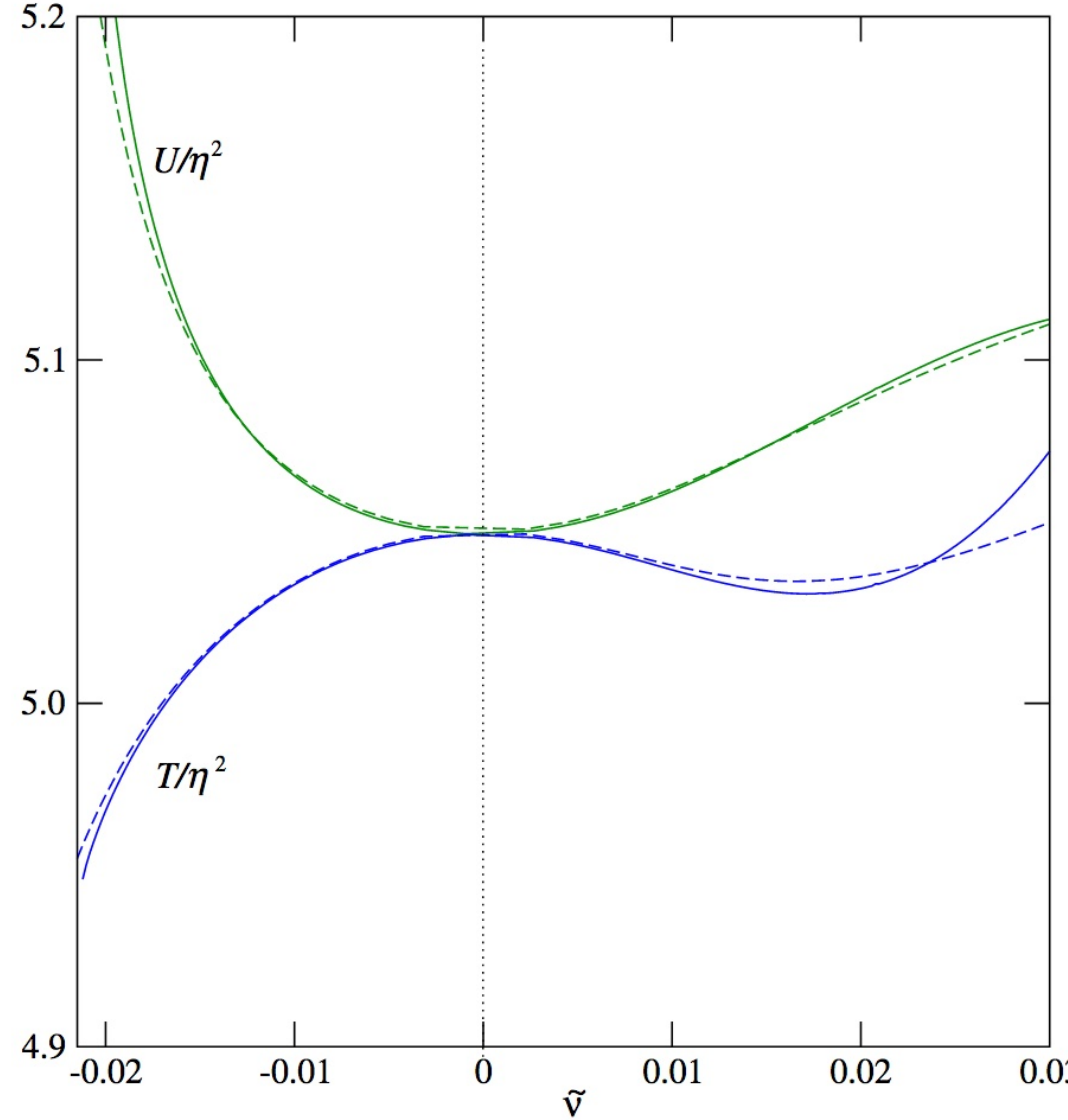
Regime	U	T	κ and χ	current
Electric	$-\Lambda$	$-\mathcal{L}$	≤ 0	timelike
Magnetic	$-\mathcal{L}$	$-\Lambda$	≥ 0	spacelike

Macroscopic perturbations (stability constraint)

$$c_{\mathrm{T}}^2 = \frac{T}{U} > 0 \qquad c_{\mathrm{L}}^2 = -\frac{\mathrm{d}T}{\mathrm{d}U} = \frac{\nu}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}\nu} > 0$$

(no spring)





Specific models

- Nambu-Goto (structureless) $\mathcal{L} = -m^2 \implies U = T$
- Small field expansion $\mathcal{L} = -m^2 + \frac{\kappa}{2} \implies \Lambda = -m^2 + \frac{\chi}{2}$ self-dual

  Subsonic equation of state $U + T = 2m^2 \implies c_{\mathrm{T}} < c_{\mathrm{L}} = 1$

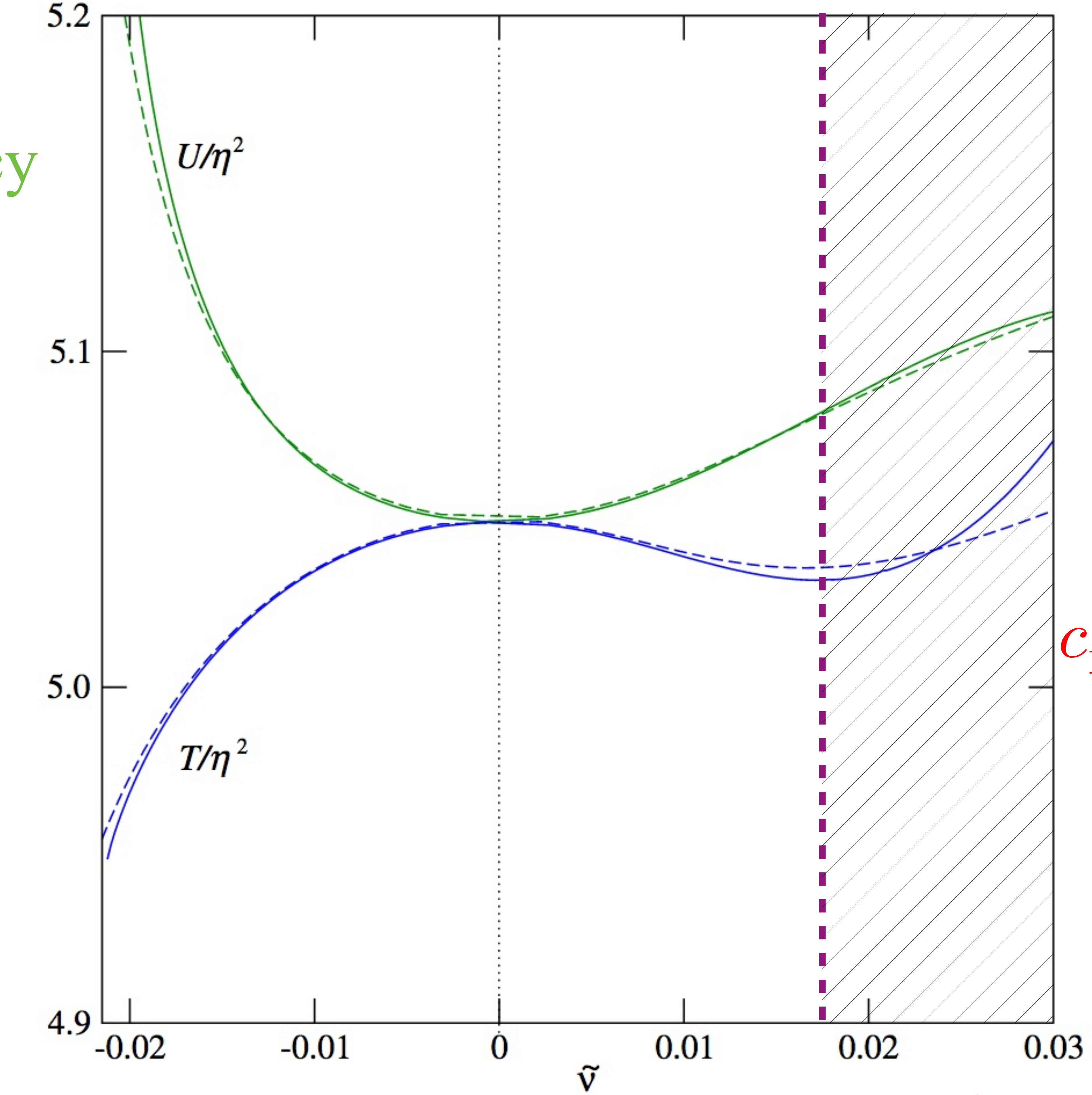
- Kaluza-Klein $\mathcal{L} = -m\sqrt{m^2 - \kappa} \implies \Lambda = -m\sqrt{m^2 - \chi}$ self-dual

  Transonic equation of state $UT = m^4 \implies c_{\mathrm{T}} = c_{\mathrm{L}} \leq 1$

- Small field expansion (higher order) $\mathcal{L} = -m^2 + \frac{\kappa}{2} + \frac{\kappa^2}{4m_*^2} + \mathcal{O}(\kappa^6)$
- Witten magnetic model $\mathcal{L} = -m^2 + \frac{\kappa}{2}\left(1 - \frac{\kappa}{m_*^2}\right)^{-1}$
- Witten electric model $\mathcal{L} = -m^2 - \frac{m_*^2}{2}\ln\left(1 - \frac{\kappa}{m_*^2}\right)$

} Supersonic equation of state $c_{\mathrm{L}} \leq c_{\mathrm{T}} \leq 1$

Phase frequency threshold



$c_{\rm L} \leq 0$

- Witten magnetic model $\mathcal{L} = -m^2 + \frac{\kappa}{2}\left(1 - \frac{\kappa}{m_*^2}\right)^{-1}$
- Witten electric model $\mathcal{L} = -m^2 - \frac{m_*^2}{2}\ln\left(1 - \frac{\kappa}{m_*^2}\right)$

non trivial structure: $$\mathcal{S}_{\mathcal{L}} = -m^2 \int \mathrm{d}^2\xi \sqrt{-\gamma}\,\mathcal{L}(\kappa)$$

$$\kappa = \gamma^{ab}\partial_a\varphi\partial_b\varphi$$

$$\Longrightarrow \quad \epsilon^2 = \frac{\boldsymbol{x}'^2}{1-\dot{\boldsymbol{x}}^2}$$

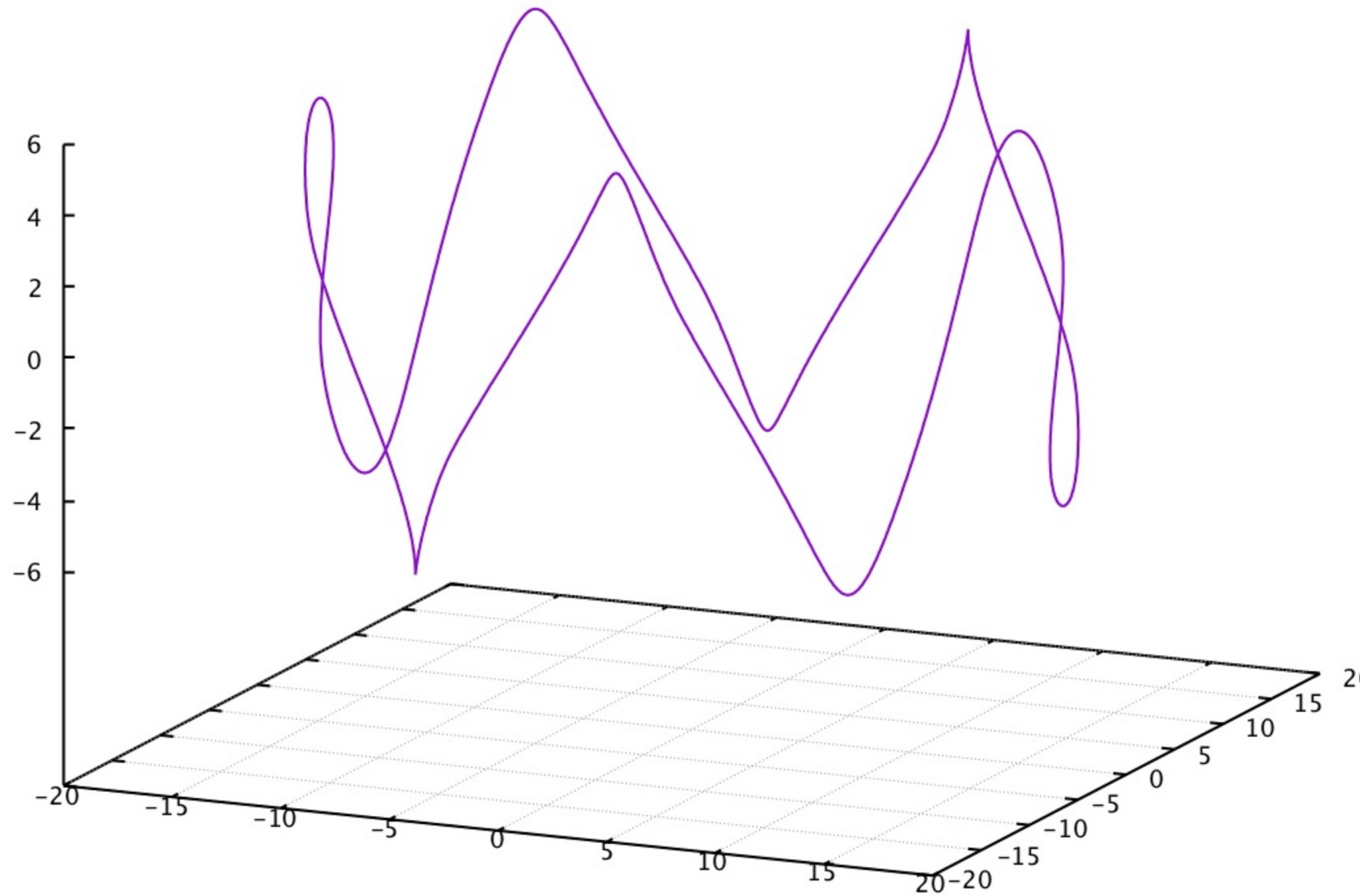
$$\frac{\partial}{\partial\tau}\left(\epsilon\dot{\varphi}\frac{\mathrm{d}\mathscr{L}}{\mathrm{d}\kappa}\right) = \frac{\partial}{\partial\sigma}\left(\frac{\varphi'}{\epsilon}\frac{\mathrm{d}\mathscr{L}}{\mathrm{d}\kappa}\right)$$

$$\frac{\partial}{\partial\tau}\left[\epsilon\left(\mathscr{L} - 2\frac{\mathrm{d}\mathscr{L}}{\mathrm{d}\kappa}\frac{\dot{\varphi}^2}{1-\dot{\boldsymbol{x}}^2}\right)\right] = -2\frac{\partial}{\partial\sigma}\left[\frac{\dot{\varphi}\varphi'}{\sqrt{\boldsymbol{x}'^2(1-\dot{\boldsymbol{x}}^2)}}\frac{\mathrm{d}\mathscr{L}}{\mathrm{d}\kappa}\right]$$

Nambu-Goto strings

$$\mathcal{S} = \int \sqrt{-\gamma}\,\mathrm{d}\sigma\mathrm{d}\tau \quad \Longrightarrow \quad \ddot{\boldsymbol{x}} = \boldsymbol{x}''$$

P. P. & B. Carter (Paris)


6
4
2
0
−2
−4
−6
−20
−15
−10
−5
0
5
10
15
20
20
15
10
5
0
−5
−10
−15
−20

Dynamics of current-carrying cosmic strings in expanding universes

Effective action $S = \int \mathcal{L}(\kappa)\sqrt{-\gamma}\,\mathrm{d}^2\sigma = -\mu_0 \int f(\kappa)\sqrt{-\gamma}\,\mathrm{d}\sigma^0\mathrm{d}\sigma^1$

Induced metric $\gamma_{ab} \equiv g_{\mu\nu}\frac{\partial X^\mu}{\partial \sigma^a}\frac{\partial X^\nu}{\partial \sigma^b} = g_{\mu\nu}X^\mu_{,a}X^\nu_{,b}$

Worldsheet $X^\mu(\sigma^a) = \left\{X^0(\sigma^0,\sigma^1), \boldsymbol{X}(\sigma^0,\sigma^1)\right\}$ Gauge choice 1: $X^0 = \tau$

Background metric $\mathrm{d}s^2_{\mathrm{FLRW}} = a^2(\tau)\left(\mathrm{d}\tau^2 - \mathrm{d}\boldsymbol{x}^2\right) = \mathrm{d}t^2 - a^2(t)\mathrm{d}\boldsymbol{x}^2$

Gauge choice 2: $\frac{\partial \boldsymbol{X}}{\partial \tau}\cdot\frac{\partial \boldsymbol{X}}{\partial \sigma} \equiv \dot{\boldsymbol{X}}\cdot\boldsymbol{X}' = 0 \implies \gamma_{ab} = \mathrm{diag}\left[a^2\left(1-\dot{\boldsymbol{X}}^2\right), -a^2\boldsymbol{X}'^2\right]$

$\implies$ State parameter $\kappa = \frac{\dot{\varphi}^2}{a^2\left(1-\dot{\boldsymbol{X}}^2\right)} - \frac{\varphi'^2}{a^2\boldsymbol{X}'^2} \equiv q^2 - j^2$

Equations of motion

$$\partial_\tau\left(\epsilon\bar{U}\right)+\frac{\dot{a}}{a}\epsilon\left[\left(\bar{U}+\bar{T}\right)\dot{\boldsymbol{X}}^2+\bar{U}-\bar{T}\right]=\partial_\sigma\Phi$$

$$\ddot{\boldsymbol{X}}\epsilon\bar{U}+\frac{\dot{a}}{a}\epsilon\left(\bar{U}+\bar{T}\right)\left(1-\dot{\boldsymbol{X}}^2\right)\dot{\boldsymbol{X}}=\partial_\sigma\left(\frac{\bar{T}}{\epsilon}\boldsymbol{X}'\right)+2\Phi\dot{\boldsymbol{X}}'+\boldsymbol{X}'\left(\dot{\Phi}+2\frac{\dot{a}}{a}\Phi\right)$$

$$\partial_\tau\left(f_\kappa a\sqrt{q^2\boldsymbol{X}'^2}\right)=\partial_\sigma\left[f_\kappa a\sqrt{j^2(1-\dot{\boldsymbol{X}}^2)}\right]$$

$$\boxed{\epsilon^2=\frac{\boldsymbol{X}'^2}{1-\dot{\boldsymbol{X}}^2}}$$

Averaging process

$$E = a\mu_0 \int \bar{U}\epsilon \, d\sigma$$ Energy

$$E_0 = a\mu_0 \int \epsilon \, d\sigma$$ Bare energy

$$\langle \mathcal{O} \rangle \equiv \frac{\int \mathcal{O}\epsilon \, d\sigma}{\int \epsilon \, d\sigma}$$ General variable

Integrated state parameter

$$K = Q^2 - J^2$$

Total charge $Q^2 \equiv \langle q^2 \rangle$ and current $J^2 \equiv \langle j^2 \rangle$

RMS velocity $v \equiv \sqrt{\langle \dot{\boldsymbol{X}}^2 \rangle}$

General assumptions / hypothesis

Uncorrelated variables

$$\langle \mathcal{F}(\mathcal{O})\rangle \approx \mathcal{F}(\langle \mathcal{O}\rangle)$$

Vanishing boundary terms

$$\int \partial_\sigma \left\{\mathcal{F}\left[\boldsymbol{X}(\sigma,\tau)\right]\right\} \mathrm{d}\sigma \;\rightarrow\; \oint \partial_\sigma \left\{\mathcal{F}\left[\boldsymbol{X}(\sigma,\tau)\right]\right\} \mathrm{d}\sigma \approx 0$$

+ Brownian string network $E = \dfrac{\mu_0 V}{L_{\mathrm{c}}^2 a^2} \iff E_0 = \dfrac{\mu_0 V}{\xi_{\mathrm{c}}^2 a^2}$

$$E = E_0\langle f - 2q^2 f_\kappa\rangle \implies \frac{E}{E_0} = F - 2Q^2 F' \qquad F(K) \equiv \langle f(\kappa)\rangle \quad F' \equiv \langle f_\kappa\rangle \quad F'' \equiv \langle f_{\kappa\kappa}\rangle$$

Equations of motion

$$\frac{\mathrm{d}L_\mathrm{c}}{\mathrm{d}\tau} = \frac{\dot{a}}{a}\frac{L_\mathrm{c}}{F - 2Q^2F'}\left\{v^2\left[F - \left(Q^2 - J^2\right)F'\right] - \left(Q^2 + J^2\right)F'\right\}$$

$$\frac{\mathrm{d}v}{\mathrm{d}\tau} = \frac{\left(1 - v^2\right)}{F - 2Q^2F'}\left\{\frac{k(v)}{L_\mathrm{c}\sqrt{F - 2Q^2F'}}\left(F + 2J^2F'\right) - 2v\frac{\dot{a}}{a}\left[F - \left(Q^2 - J^2\right)F'\right]\right\}$$

$$\frac{\mathrm{d}J^2}{\mathrm{d}\tau} = 2J^2\left[\frac{vk(v)}{L_\mathrm{c}\sqrt{F - 2Q^2F'}} - \frac{\dot{a}}{a}\right]$$

$$\frac{\mathrm{d}Q^2}{\mathrm{d}\tau} = 2Q^2\frac{F' + 2J^2F''}{F' + 2Q^2F''}\left[\frac{vk(v)}{L_\mathrm{c}\sqrt{F - 2Q^2F'}} - \frac{\dot{a}}{a}\right]$$

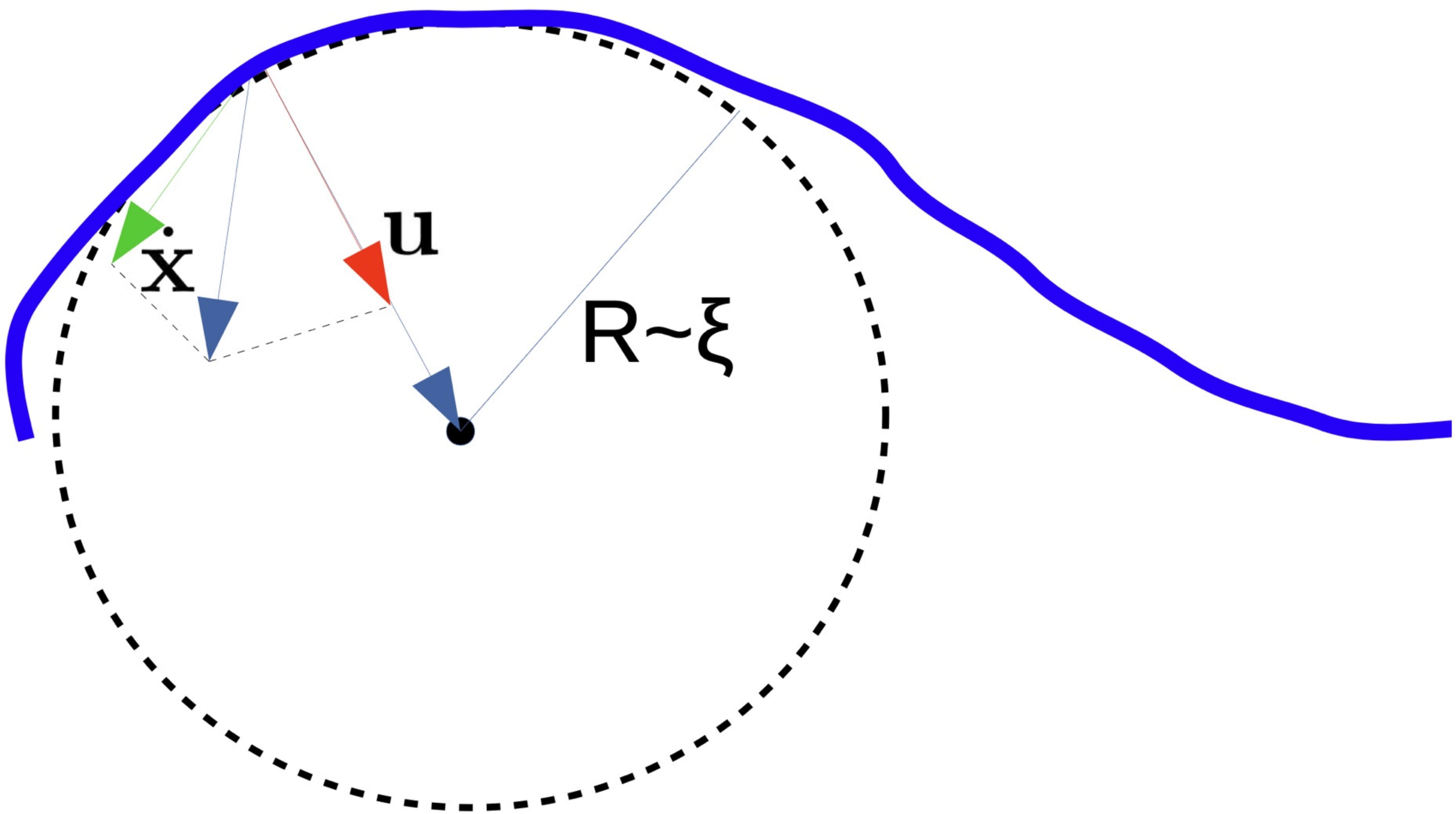
Chirality

$$K = Q^2 - J^2$$

Charge

$$Y = \frac{1}{2}(Q^2 + J^2)$$

Momentum parameter: $k(v) \equiv \frac{\left\langle (1-\dot{\boldsymbol{X}}^2)(\dot{\boldsymbol{X}}\cdot\boldsymbol{u})\right\rangle}{v(1-v^2)}$

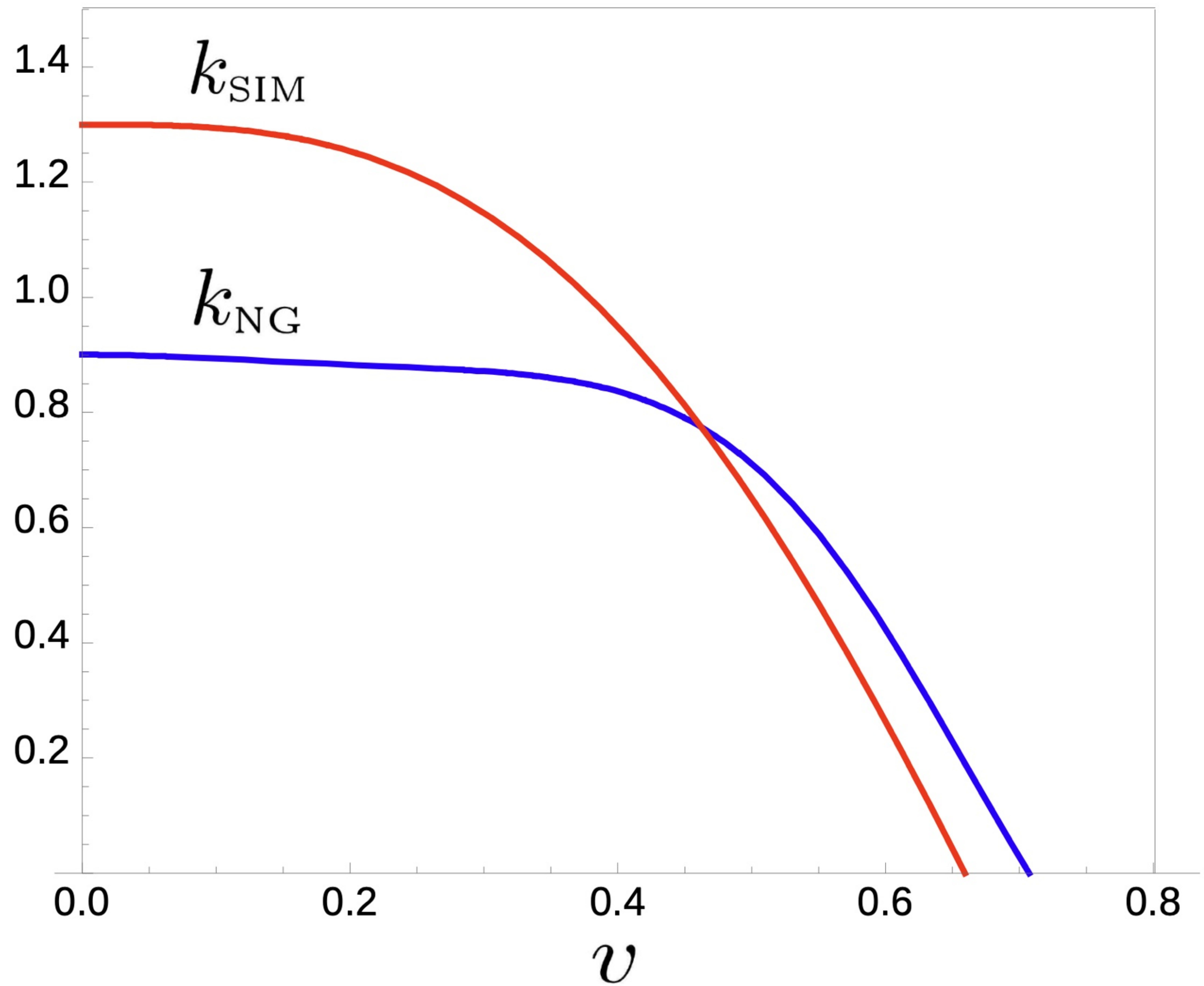


Momentum parameter: $$k(v) \equiv \frac{\left\langle (1-\dot{\boldsymbol{X}}^2)(\dot{\boldsymbol{X}}\cdot\boldsymbol{u}) \right\rangle}{v(1-v^2)}$$

From Nambu-Goto network simulations $$k_{\rm NG}(v) = \frac{2\sqrt{2}}{\pi}\frac{1-8v^6}{1+8v^6}\,(1-v^2)\left(1+2\sqrt{2}v^3\right)$$

From Abelian-Higgs simulations $$k_{\rm SIM}(v) = k_0\frac{1-(\alpha v^2)^\beta}{1+(\alpha v^2)^\beta}$$

Momentum parameter: $k(v) \equiv \frac{\left\langle (1-\dot{\boldsymbol{X}}^2)(\dot{\boldsymbol{X}}\cdot\boldsymbol{u}) \right\rangle}{v(1-v^2)}$



Phenomenological parameter:

- loop chopping efficiency $$\left.\frac{\mathrm{d}E_0}{\mathrm{d}\tau}\right|_{\mathrm{loops}} = -\, c\, v \frac{E_0}{\xi_{\mathrm{c}}}$$

($c \simeq 0.23$ in ordinary "VOS" model)

- current chopping efficiency $$\left.\frac{\mathrm{d}E}{\mathrm{d}\tau}\right|_{\mathrm{loops}} = -\, g\, cv \frac{E}{\xi_{\mathrm{c}}}$$

- charge leakage $$\left.\frac{\mathrm{d}Y}{\mathrm{d}\tau}\right|_{\mathrm{leakage}} = -\, A\, \frac{Y}{\xi_{\mathrm{c}}} = -\, A\, \frac{Y}{L_{\mathrm{c}}\sqrt{F - 2Q^2F'}} \to \frac{Y}{L_{\mathrm{c}}\sqrt{1+Y}}$$

Universe expansion: $$a(\tau) = a_{\mathrm{eq}} \left[2\left(\frac{\tau}{\tau_{\mathrm{eq}}}\right) + \left(\frac{\tau}{\tau_{\mathrm{eq}}}\right)^2 \right]$$

Scaling solution $L_{\mathrm{c}} = \zeta\tau$ with $\dot{\zeta} = 0$

$$\dot{v} = 0$$

$$\dot{Y} = \dot{K} = 0$$

Linear regime $F(K) = 1 - \frac{\kappa_0}{2} K$

$$\dot{\zeta}\tau = \frac{v^2 + Y}{1+Y}\frac{\dot{a}}{a}\zeta + \frac{gcv(1+Y) + AY}{2(1+Y)^{3/2}} - \zeta$$

$$\dot{v}\tau = \frac{1-v^2}{1+Y}\left[\frac{k\,(1-Y)}{\zeta\sqrt{1+Y}} - 2v\frac{\dot{a}}{a}\right]$$

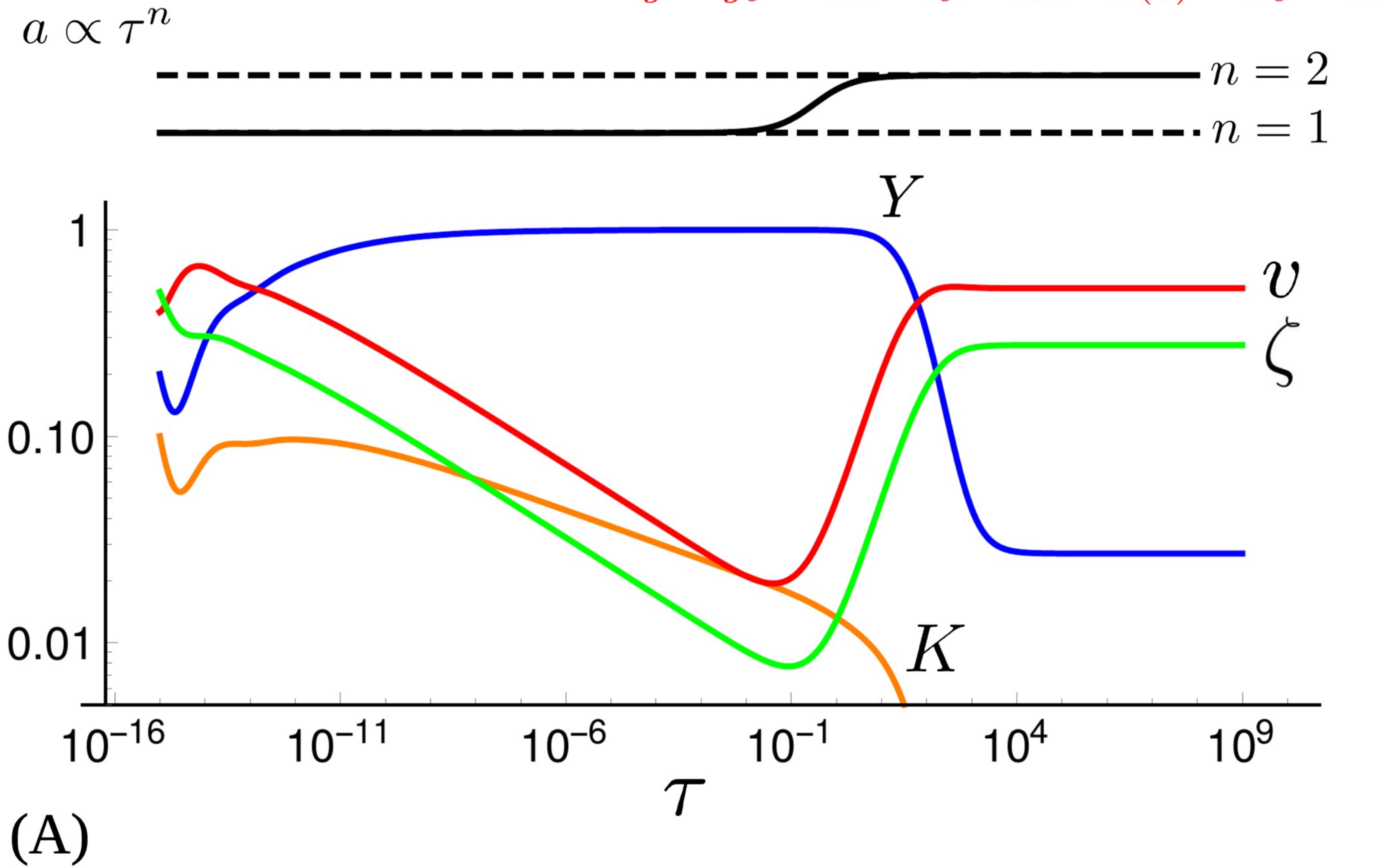
$$\dot{Y}\tau = 2Y\left(\frac{vk}{\zeta\sqrt{1+Y}} - \frac{\dot{a}}{a}\right) - \frac{vc(g-1)}{\zeta}\sqrt{1+Y} - \frac{AY}{\zeta\sqrt{1+Y}}$$

$$\dot{K} = 2K\left(\frac{vk}{L_{\mathrm{c}}\sqrt{1+Y}} - \frac{\dot{a}}{a}\right) - \frac{2(1-2\rho_{\mathrm{A}})AY}{L_{\mathrm{c}}\sqrt{1+Y}} - 2\frac{v}{L_{\mathrm{c}}}c\,(g-1)\,(1-2\rho)\sqrt{1+Y}$$

Dynamical solutions

No leakage

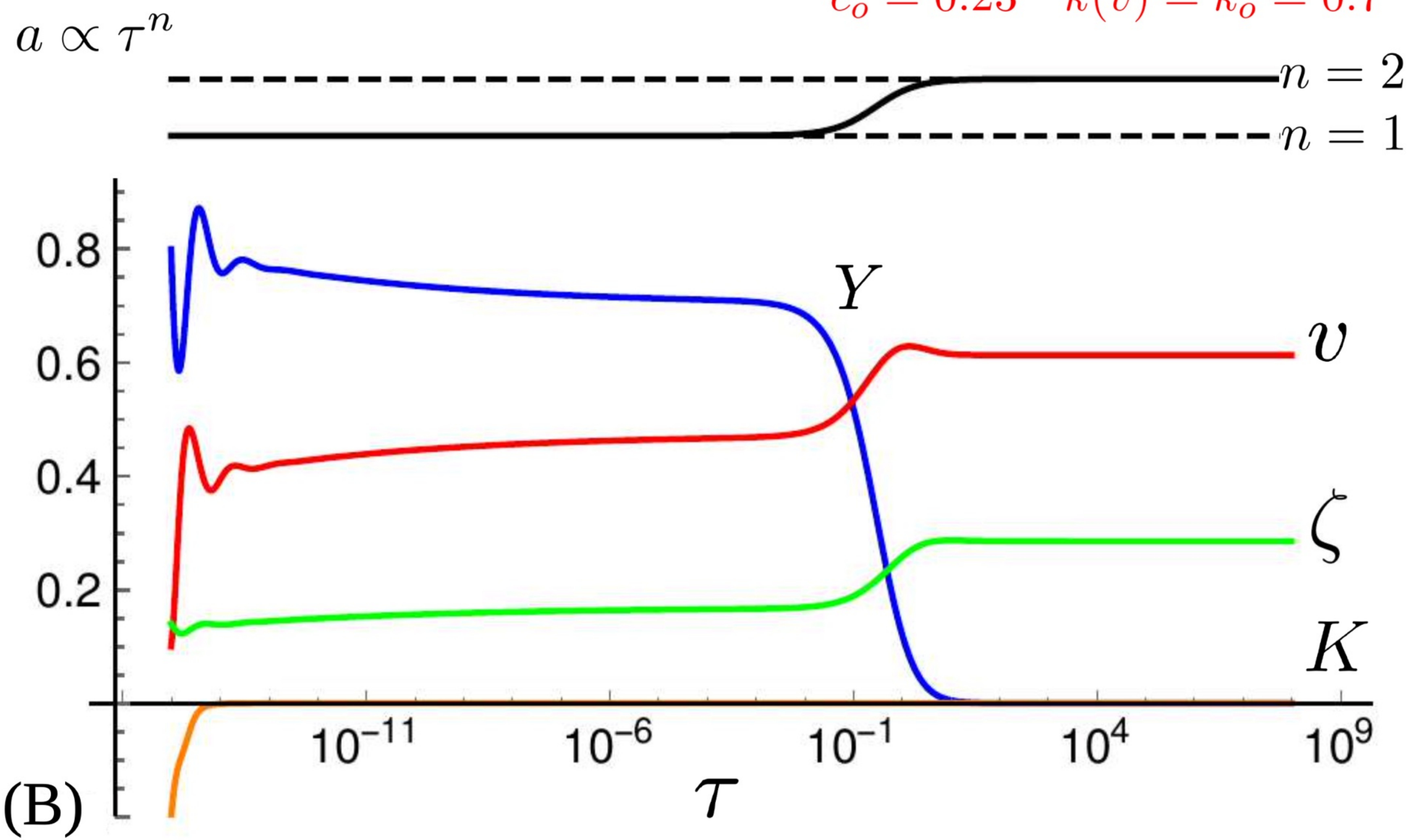
$g = g_o = 0.9 \quad c_o = 0.5 \quad k(v) = k_o = 0.6$



Dynamical solutions

No leakage $g = 1 + 2bY$

$c_o = 0.23 \quad k(v) = k_o = 0.7 \quad b = 0.6$

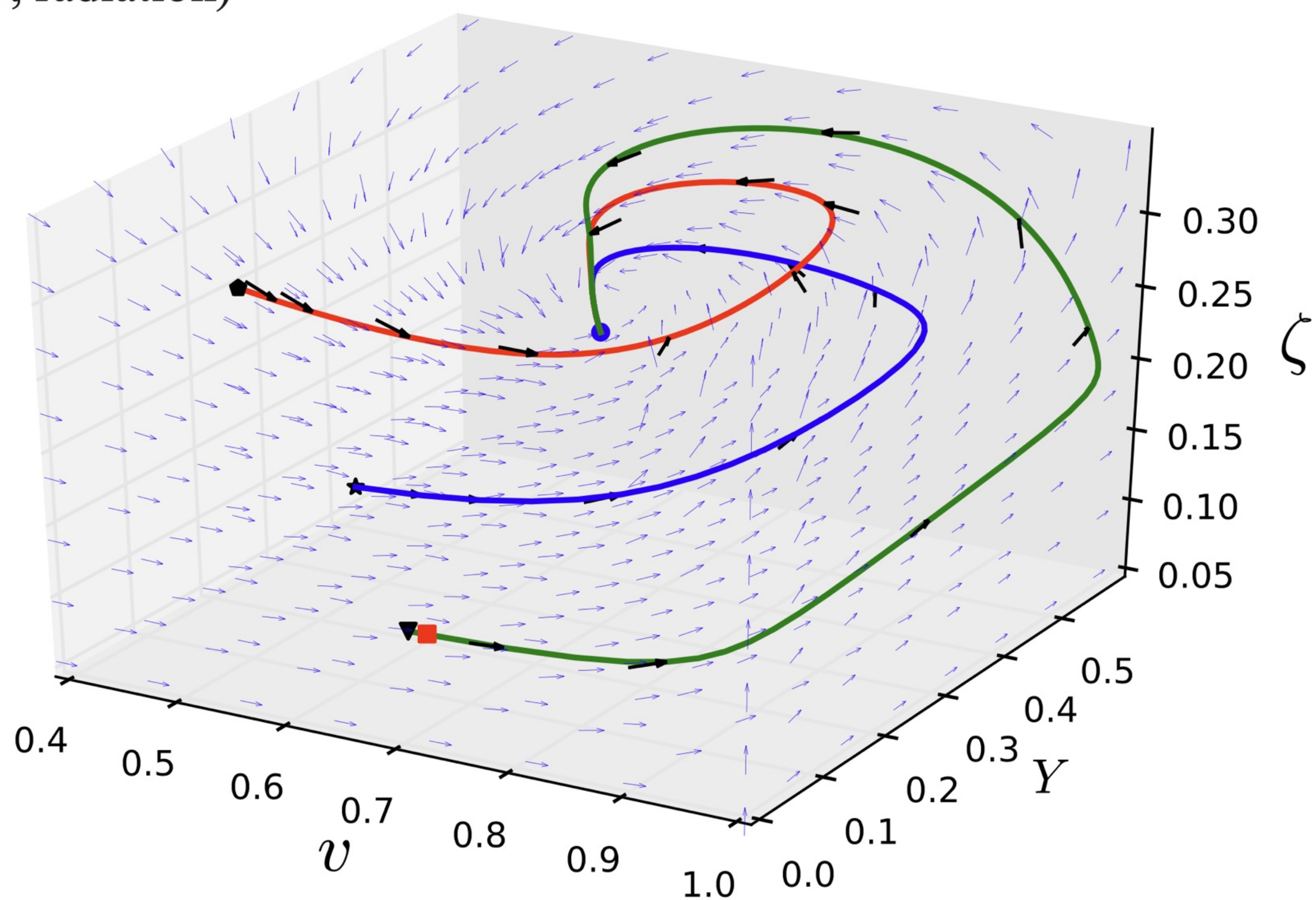


Dynamical solutions

No leakage $g = 1 + 2bY$

$c_o = 0.23 \quad k(v) = k_o = 0.6 \quad b = 0.4$

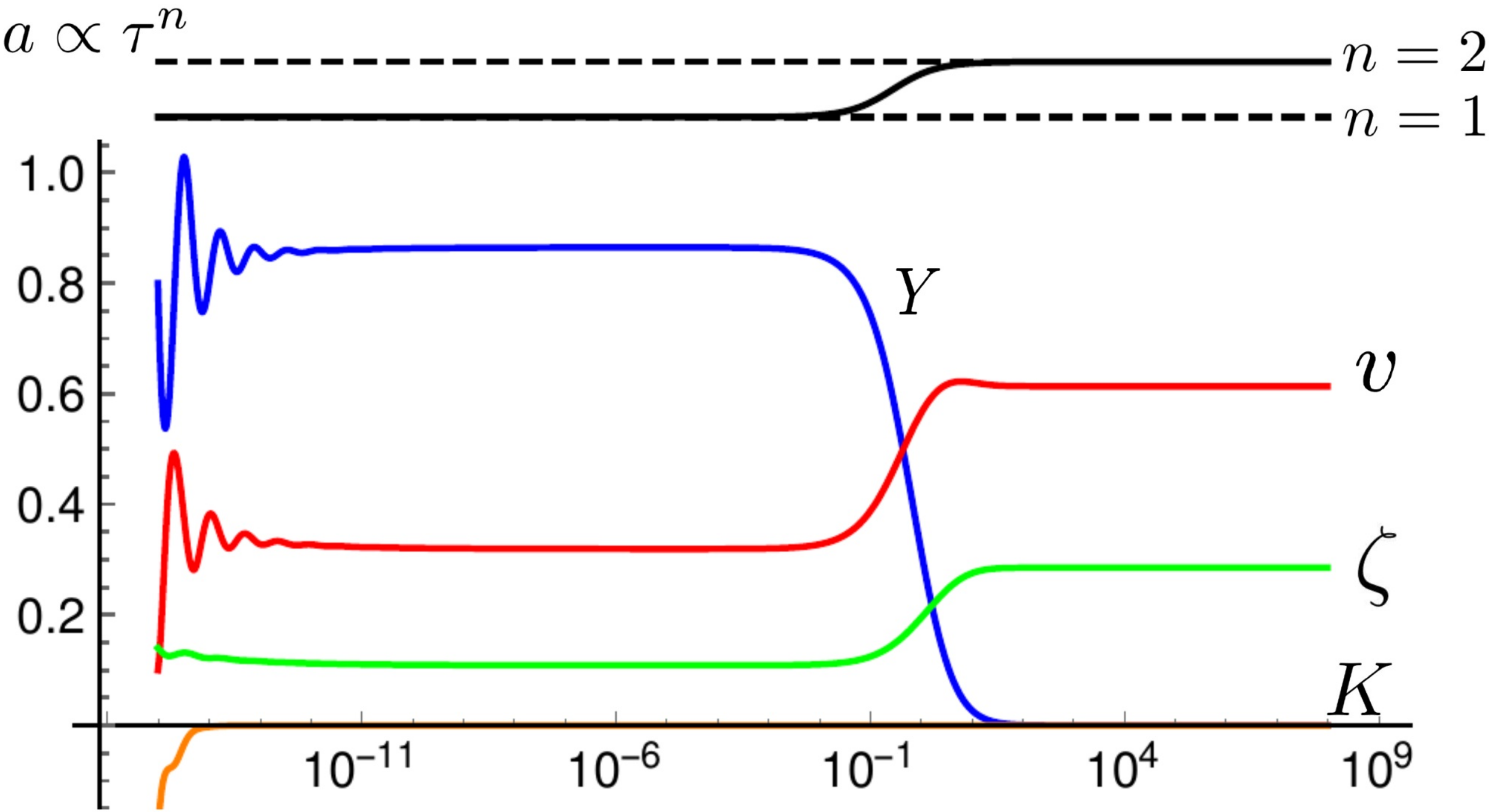
Attractor ($n = 1$, radiation)



Dynamical solutions

With leakage

$c_o = 0.23 \quad k(v) = k_o = 0.7 \quad b = 0 \quad A = 0.6$

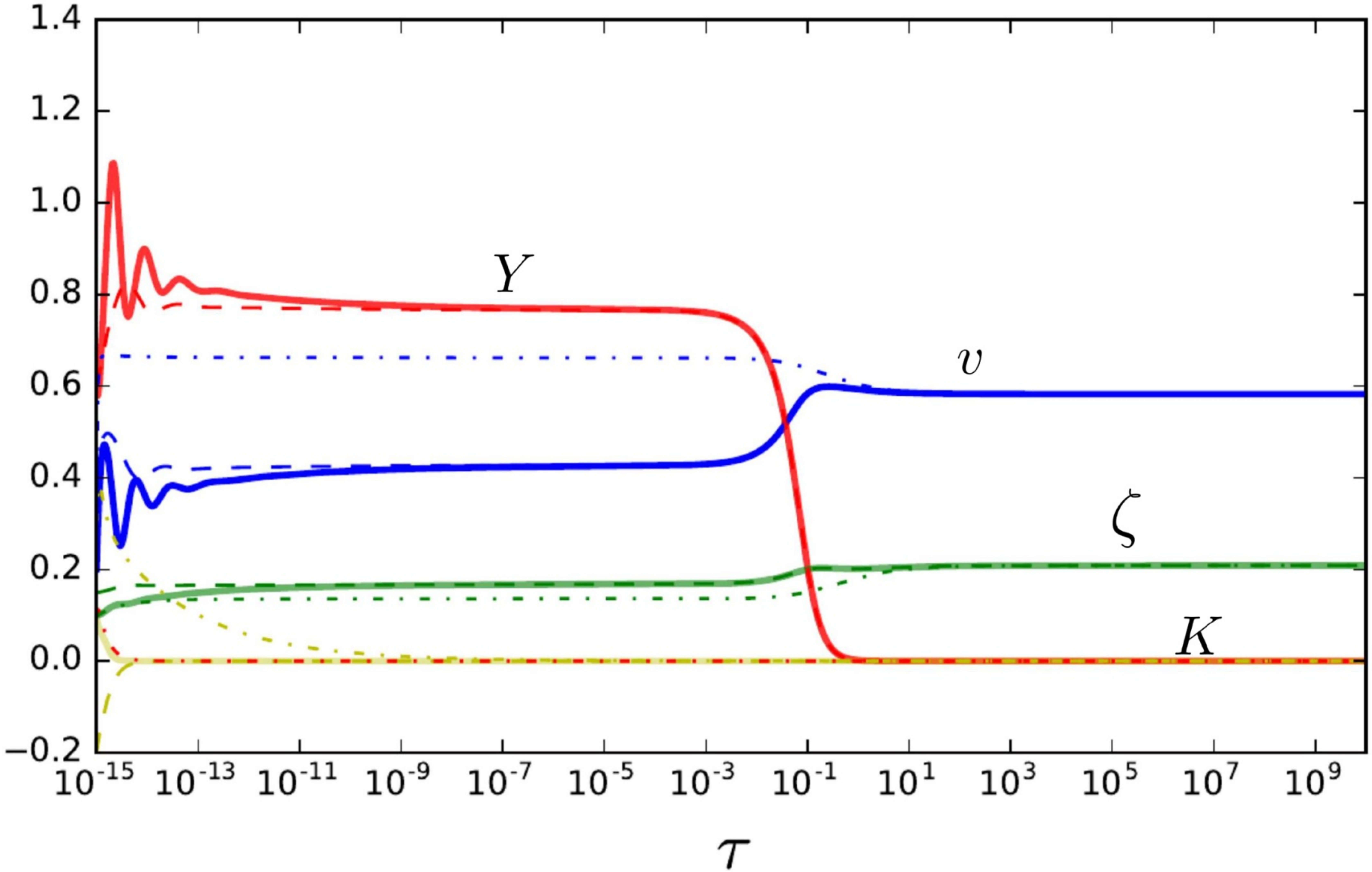


(D)

Dynamical solutions

With leakage $g = 1 + 2bY$

$c_o = 0.23 \quad b = 0 \quad A = 0.25$



Beyond the linear model

Relevant thermodynamical variables

$$Y = \frac{1}{2}\left(Q^2 + J^2\right) \quad \& \quad K = Q^2 - J^2 \Longrightarrow Q^2 = Y + K/2 \quad \& \quad J^2 = Y - K/2$$

constraint $|K| \leq 2Y$

Charge and current leakage

$$\left.\frac{\mathrm{d}Q^2}{\mathrm{d}\tau}\right|_{\mathrm{leak}} = -A\frac{Q^2}{\xi_{\mathrm{C}}}, \qquad \left.\frac{\mathrm{d}J^2}{\mathrm{d}\tau}\right|_{\mathrm{leak}} = -B\frac{J^2}{\xi_{\mathrm{C}}}$$

$$\Longrightarrow \begin{cases} \left.\dfrac{\mathrm{d}Y}{\mathrm{d}\tau}\right|_{\mathrm{leak}} = -\dfrac{1}{2\xi_{\mathrm{C}}}\left(A_+ Y + A_- \dfrac{K}{2}\right) \\ \left.\dfrac{\mathrm{d}K}{\mathrm{d}\tau}\right|_{\mathrm{leak}} = -\dfrac{1}{\xi_{\mathrm{C}}}\left(A_- Y + A_+ \dfrac{K}{2}\right) \end{cases}$$

Loop chopping bias

$$\left.\frac{\mathrm{d}Q^2}{\mathrm{d}\tau}\right|_{\mathrm{loops}} = -g_Q \frac{cv}{\xi_{\mathrm{C}}} Q^2$$

$$\left.\frac{\mathrm{d}J^2}{\mathrm{d}\tau}\right|_{\mathrm{loops}} = -g_J \frac{cv}{\xi_{\mathrm{C}}} J^2$$

Relation with linear bias

$$g = 1 - g_Q \frac{F' + 2Q^2 F''}{F - 2Q^2 F'} Q^2 - g_J \frac{F' - 2Q^2 F''}{F - 2Q^2 F'} J^2$$

$$\Longrightarrow \begin{cases} \left.\dot{Y}\right|_{\mathrm{loops}} = -\dfrac{cv}{2\xi_{\mathrm{C}}}\left(g_+ Y + g_- \dfrac{K}{2}\right) \\ \left.\dot{K}\right|_{\mathrm{loops}} = -\dfrac{cv}{\xi_{\mathrm{C}}}\left(g_- Y + g_+ \dfrac{K}{2}\right) \end{cases}$$

Modified equations of motion in terms of $\xi_C = \epsilon\tau$

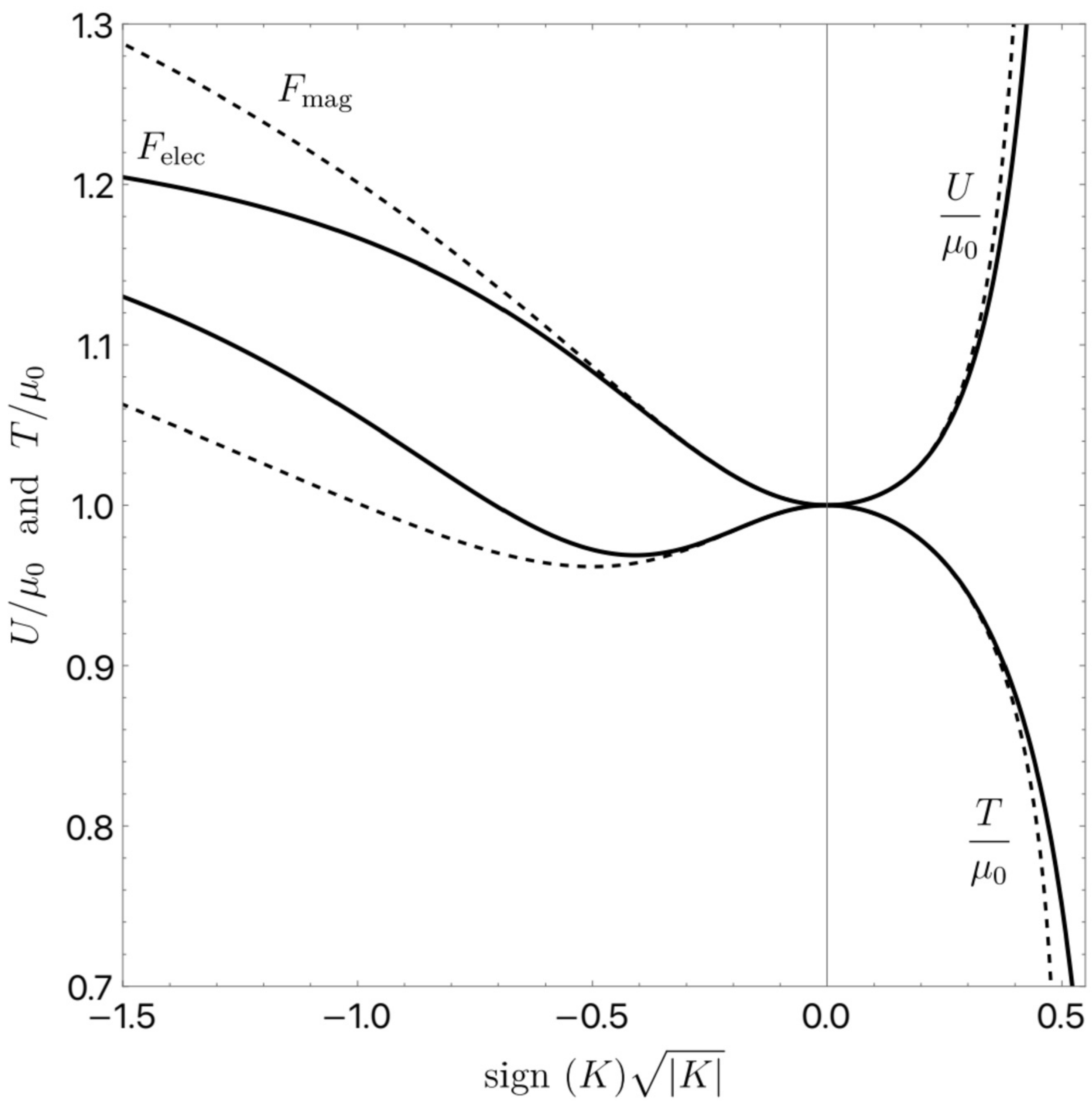
$$\left(\text{definition}:\ \xi_C = \sqrt{F - 2Q^2 F'}\, L_C =: W L_C\right)$$

$$
\begin{cases}
\dot{\epsilon}\tau = \dfrac{1}{W^2}\left[n\epsilon v^2 (F - F'K) - 2vkYF'\right] + \dfrac{1}{2}cv - \epsilon \\[2ex]
\dot{v}\tau = \dfrac{1-v^2}{W^2}\left\{\dfrac{k}{\epsilon}\left[F + 2\left(Y - \dfrac{K}{2}\right)F'\right] - 2vn(F - F'K)\right\} \\[2ex]
\dot{Y}\tau = \left(\dfrac{vk}{\epsilon} - n\right)\dfrac{2YF' + (4Y^2 - K^2)F''}{F' + (2Y+K)F''} - \dfrac{cv}{2\epsilon}\left(g_+ Y + g_- \dfrac{K}{2}\right) - \dfrac{2A_+ Y + A_- K}{4\epsilon} \\[2ex]
\dot{K}\tau = 2\left(\dfrac{vk}{\epsilon} - n\right)\dfrac{F'K}{F' + (2Y+K)F''} - \dfrac{cv}{\epsilon}\left(g_- Y + g_+ \dfrac{K}{2}\right) - \dfrac{2A_- Y + A_+ K}{2\epsilon}
\end{cases}
$$

Enhancement of charge / current loss for $Y \gtrsim Y_{\rm cr}$:

$$A_\pm(Y) = \frac{A_{\rm const}}{1 - {\rm e}^{-(Y - Y_{\rm cr})^2}}$$

$$Y_{\rm cr} = \frac{2}{3\alpha}$$

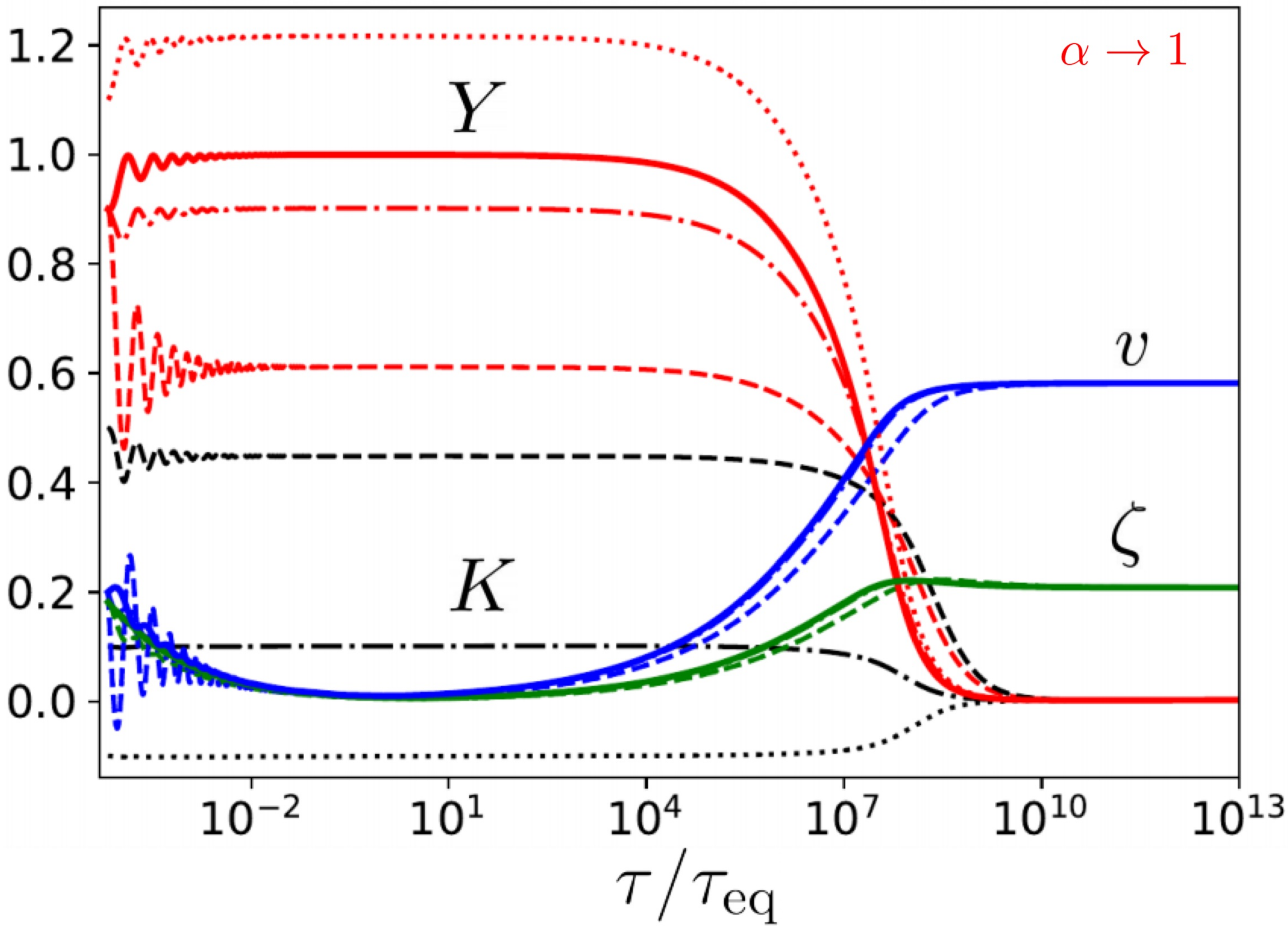


$$F_{\rm mag}(K) = 1 - \frac{1}{2}\frac{K}{1 - \alpha K} \quad \text{for} \quad K \le 0$$

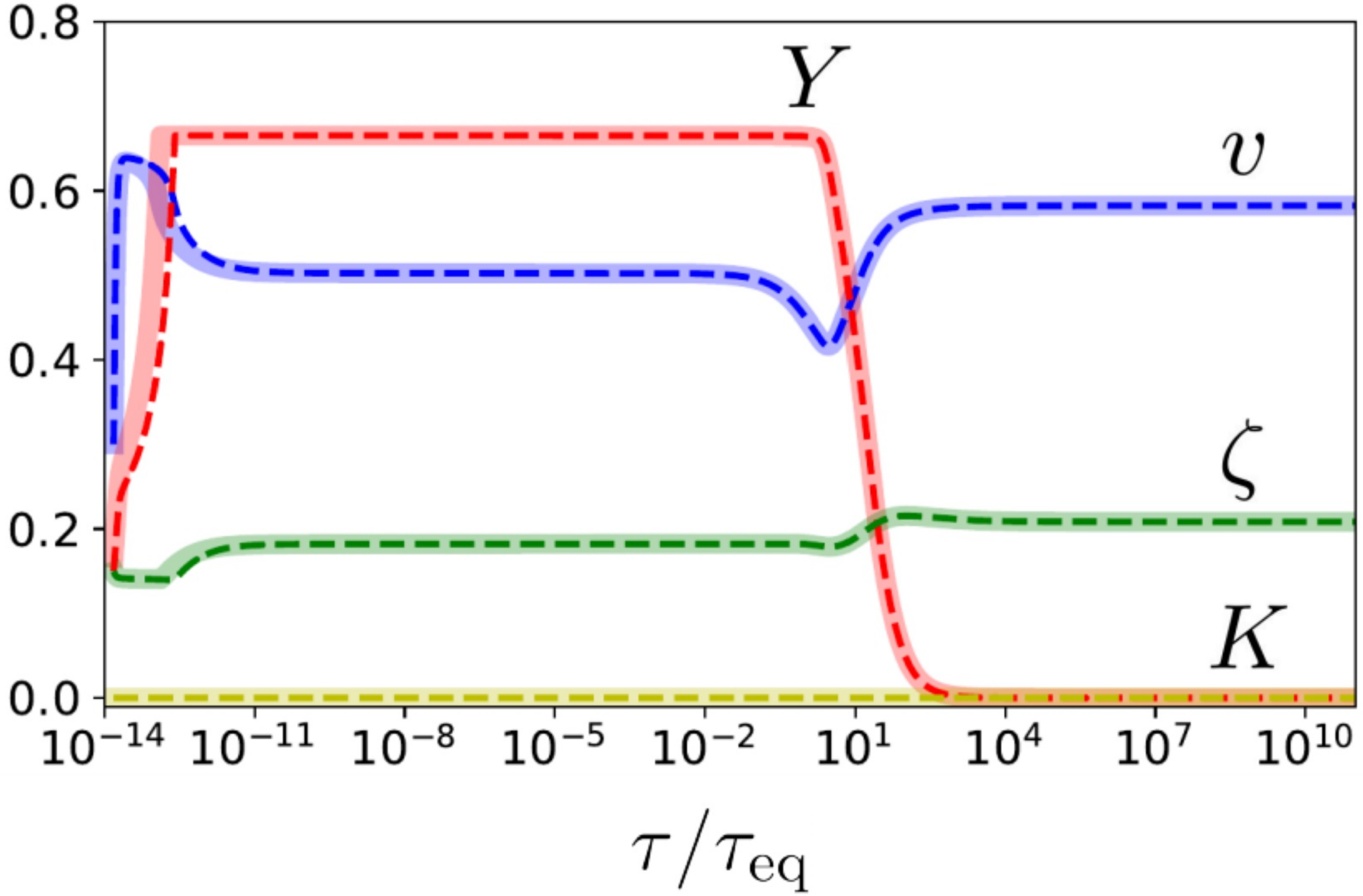
$$F_{\rm elec}(K) = 1 + \frac{\ln(1 - 2\alpha K)}{4\alpha} \quad \text{for} \quad K \ge 0$$

$$\alpha = \left(\frac{m_{\rm H}}{m_\sigma}\right)^2 \gg 1$$

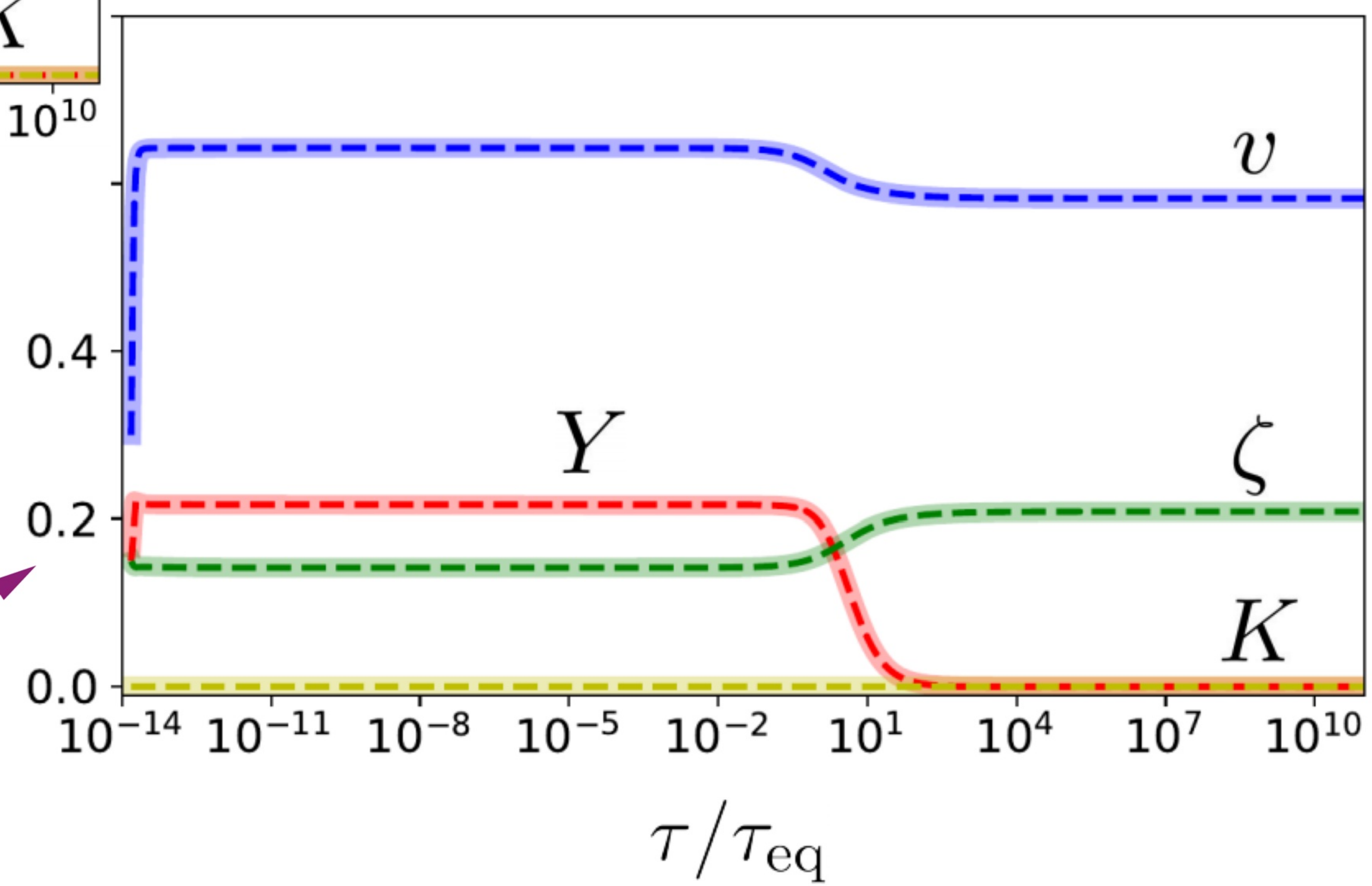
Case with no leakage: ***frozen network***



Leakage and no bias: *linear regime*



$\alpha \to 1$



$\alpha \to 3$

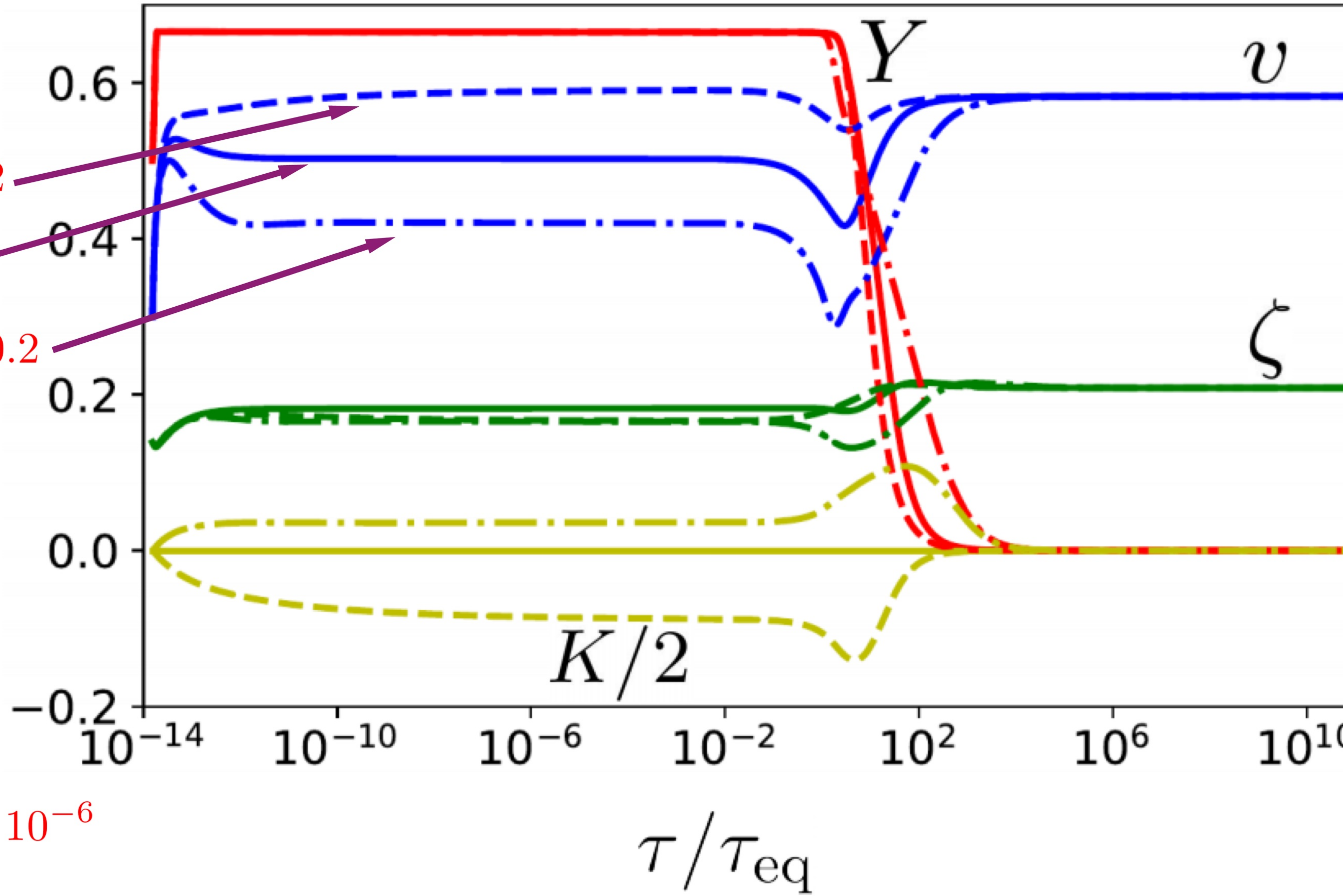
$g_{\pm} = 0$

$A_{-} = 0$

$A_{\mathrm{const}} = 10^{-6}$

Leakage and bias: [*full Witten regime*]

$\alpha \to 1$



$g_- = 0.2$

$g_- = 0$

$g_- = -0.2$

$g_+ = 0$

$A_- = 0$

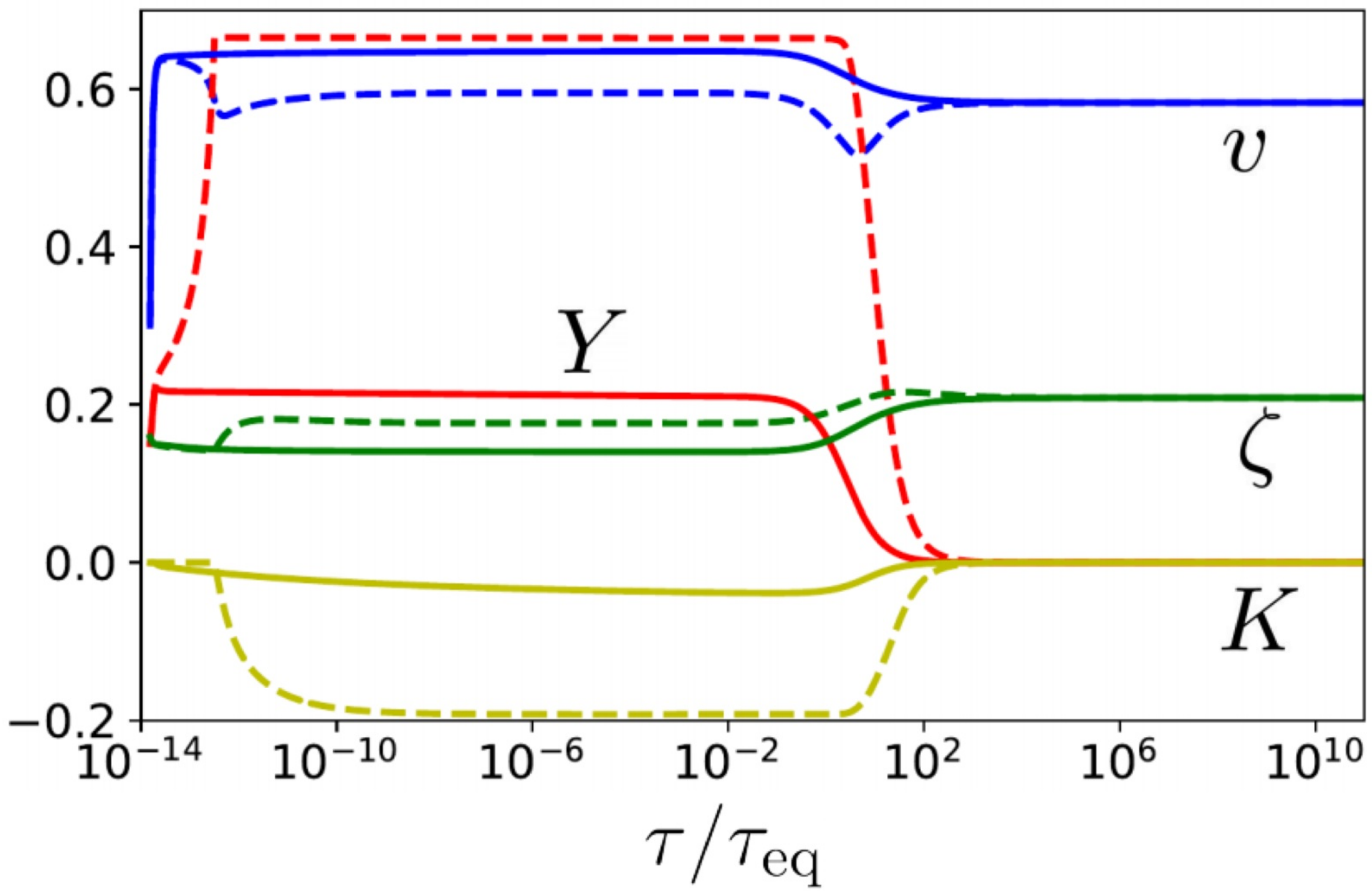
$A_{\rm const} = 10^{-6}$

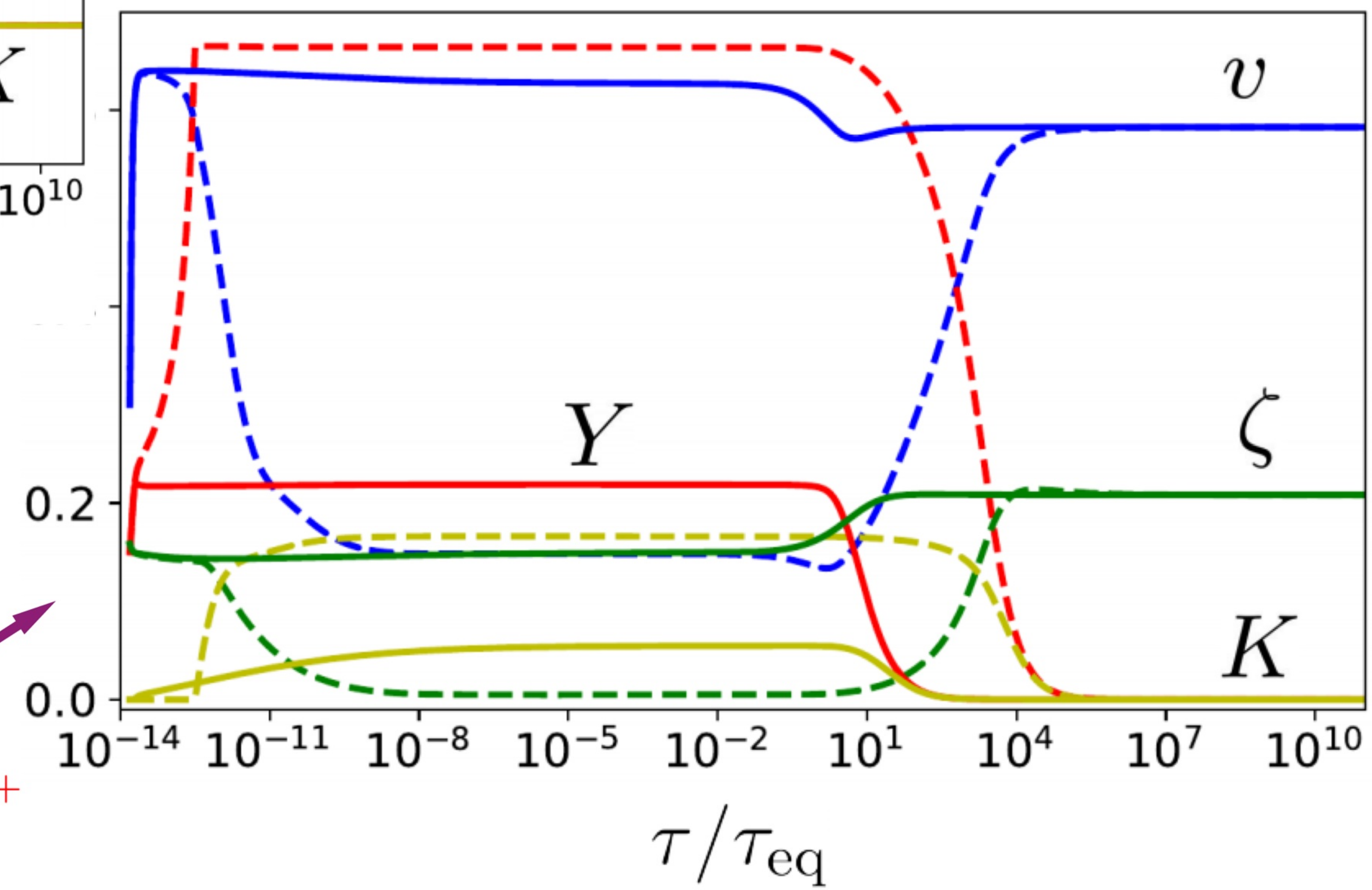
Leakage and bias: ***full Witten regime***

$\alpha \to 1$ (dashed)

$\alpha \to 3$ (solid)

$A_- = 0.1 A_+$





$g_\pm = 0$

$A_{\rm const} = 10^{-6}$

$A_- = -0.1 A_+$

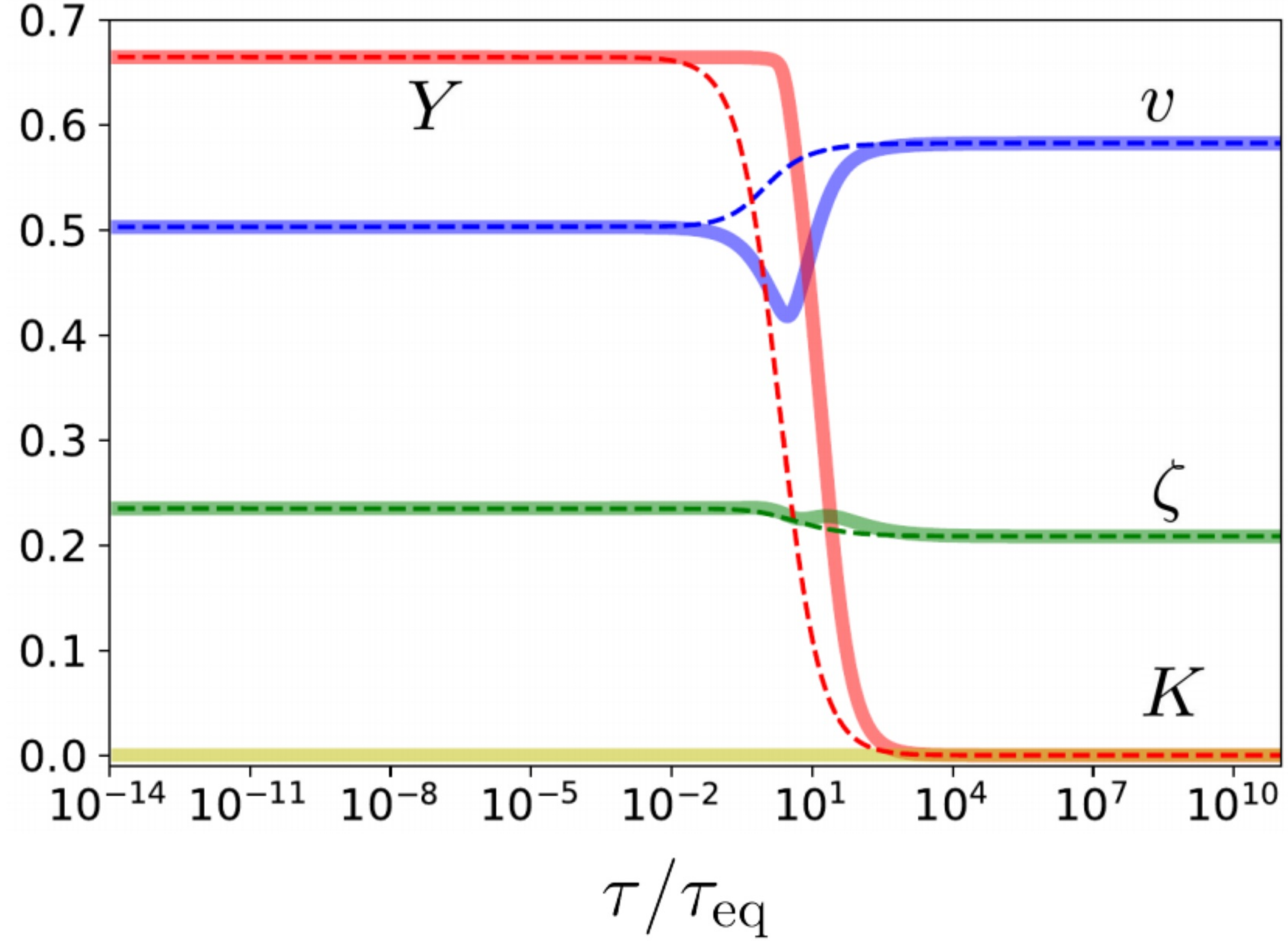
General behaviour of the network at this stage

- No leakage $\Longrightarrow$ ***frozen network***
- Matter domination era $\Longrightarrow$ Nambu-Goto evolution
- Radiation domination era $\Longrightarrow$ non trivial charge / current ``**linear**'' scaling
- Radiation domination era $\Longrightarrow$ velocity & density $Y-$ dependent scaling

$$\left.\begin{aligned}
\frac{d\xi_C}{d\tau} &= \frac{\xi_C v^2}{1+Y}\frac{\dot a}{a} + \frac{cv}{2} + \frac{Y}{1+Y} v k(v) \\
\frac{dv}{d\tau} &= \frac{(1-v^2)}{1+Y}\left[\frac{k(v)\,(1-Y)}{\xi_C} - 2v\frac{\dot a}{a}\right] \\
\frac{dY}{d\tau} &= 2Y\left[\frac{vk(v)}{\xi_C} - \frac{\dot a}{a}\right] - \frac{Y}{\xi_C}A(Y) \\
\frac{dK}{d\tau} &= 2K\left[\frac{vk(v)}{\xi_C} - \frac{\dot a}{a}\right] - \frac{K}{\xi_C}A(Y)
\end{aligned}\right\} \longrightarrow
\left\{\begin{aligned}
\frac{d\xi_C}{d\tau} &= \frac{\xi_C v^2}{1+Y_f}\frac{\dot a}{a} + \frac{cv}{2} + \frac{Y_f}{1+Y_f} v k(v) \\
\frac{dv}{d\tau} &= \frac{(1-v^2)}{1+Y_f}\left[\frac{k(v)\,(1-Y_f)}{\xi_C} - 2v\frac{\dot a}{a}\right]
\end{aligned}\right.$$

+ Heaviside $Y-$ scaling

$$Y_f = Y_{SC}\left(2 - \frac{\dot a}{a}\tau\right)\Theta\left(2 - \frac{\dot a}{a}\tau\right)$$



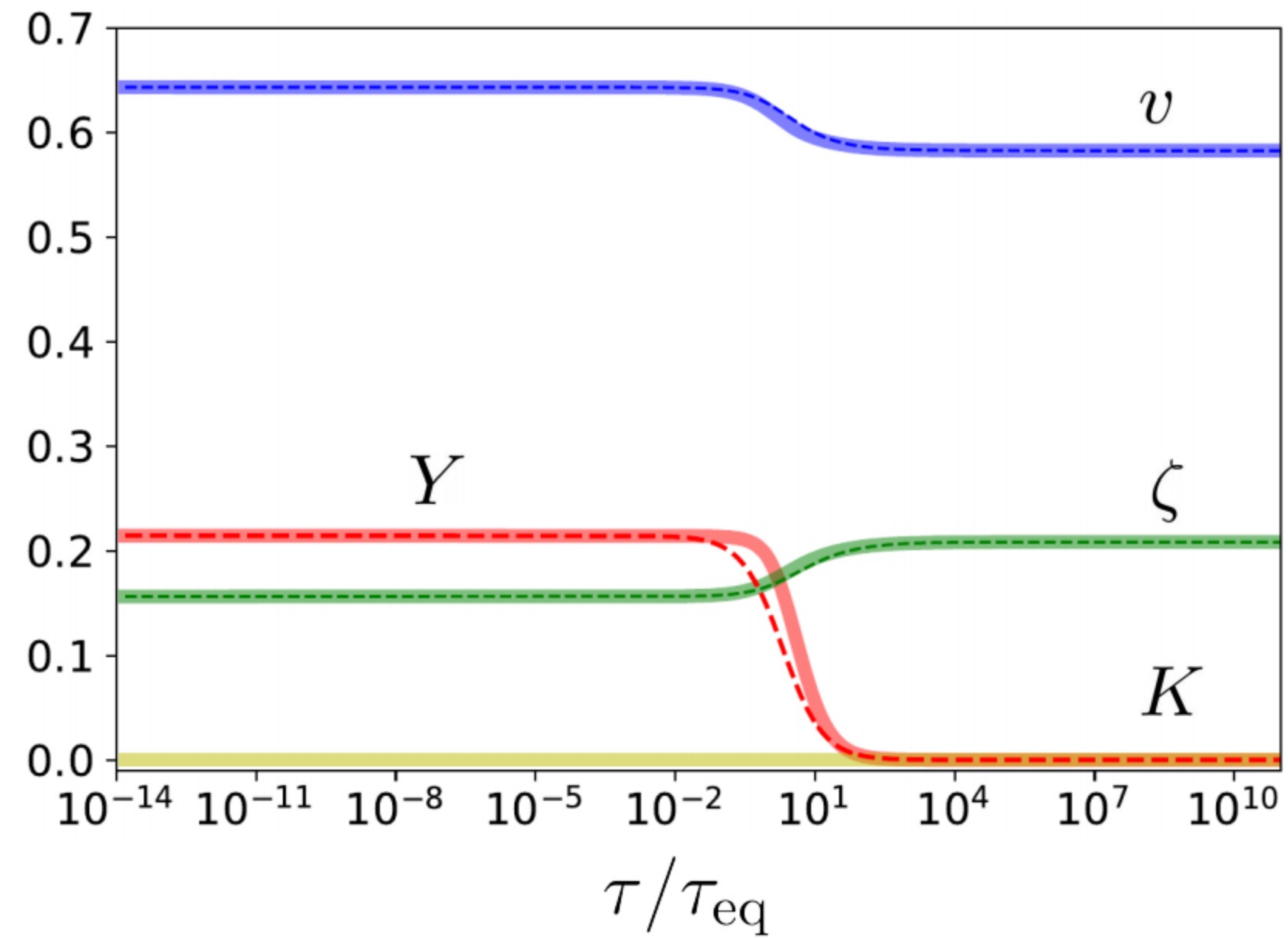
Full system **:** solid line

$\alpha \to 1$

Heaviside approximation **:** dashed line

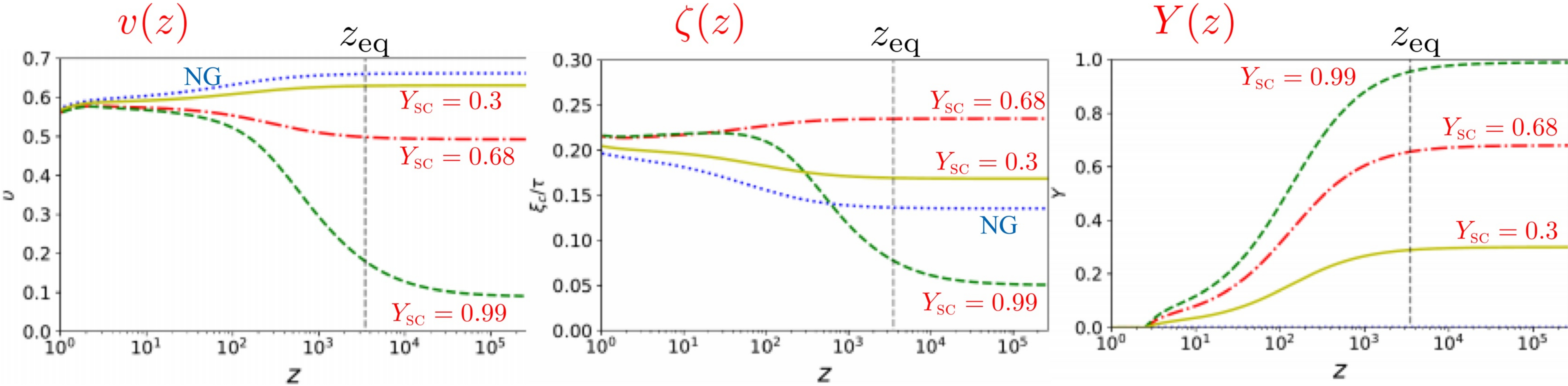
One-parameter ($Y_{\rm SC}^{\rm rad}$) model

$\alpha \to 3$

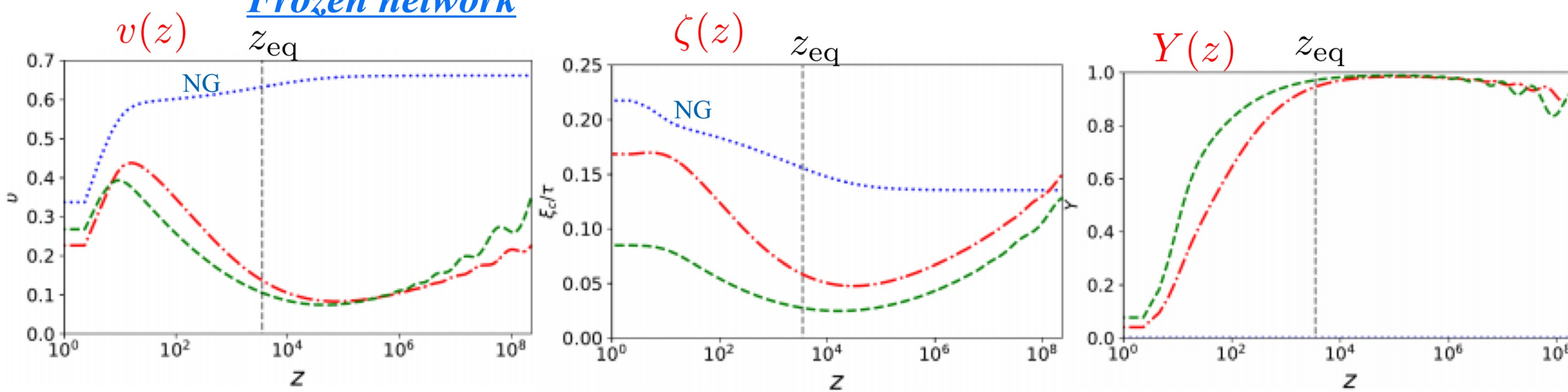


Observational setting (redshift)

Radiation scaling



Frozen network



CMBACT = CMB from ACTive sources…

$$T^{\mu\nu}\left(x^\lambda\right) = \frac{\mu_0}{\sqrt{-g}} \int \mathrm{d}^2\sigma \sqrt{-\gamma}\,\delta^{(4)}\left[x^\lambda - X^\lambda(\sigma,\tau)\right]\left[(1+Y)\tilde{u}^\mu\tilde{u}^\nu - (1-Y)\tilde{v}^\mu\tilde{v}^\nu - Y\left(\tilde{u}^\mu\tilde{v}^\nu + \tilde{v}^\mu\tilde{u}^\nu\right)\right]$$

$$X^\mu = x_0^\mu + \sigma \hat{X}_1^\mu + \tau \hat{X}_2^\mu \qquad \tilde{u}^\mu \to \frac{\dot{X}^\mu}{\sqrt{\dot{X}^2}} \qquad \tilde{v}^\mu \to \frac{X'^\mu}{\sqrt{-X'^2}}$$

Straight segments approximation

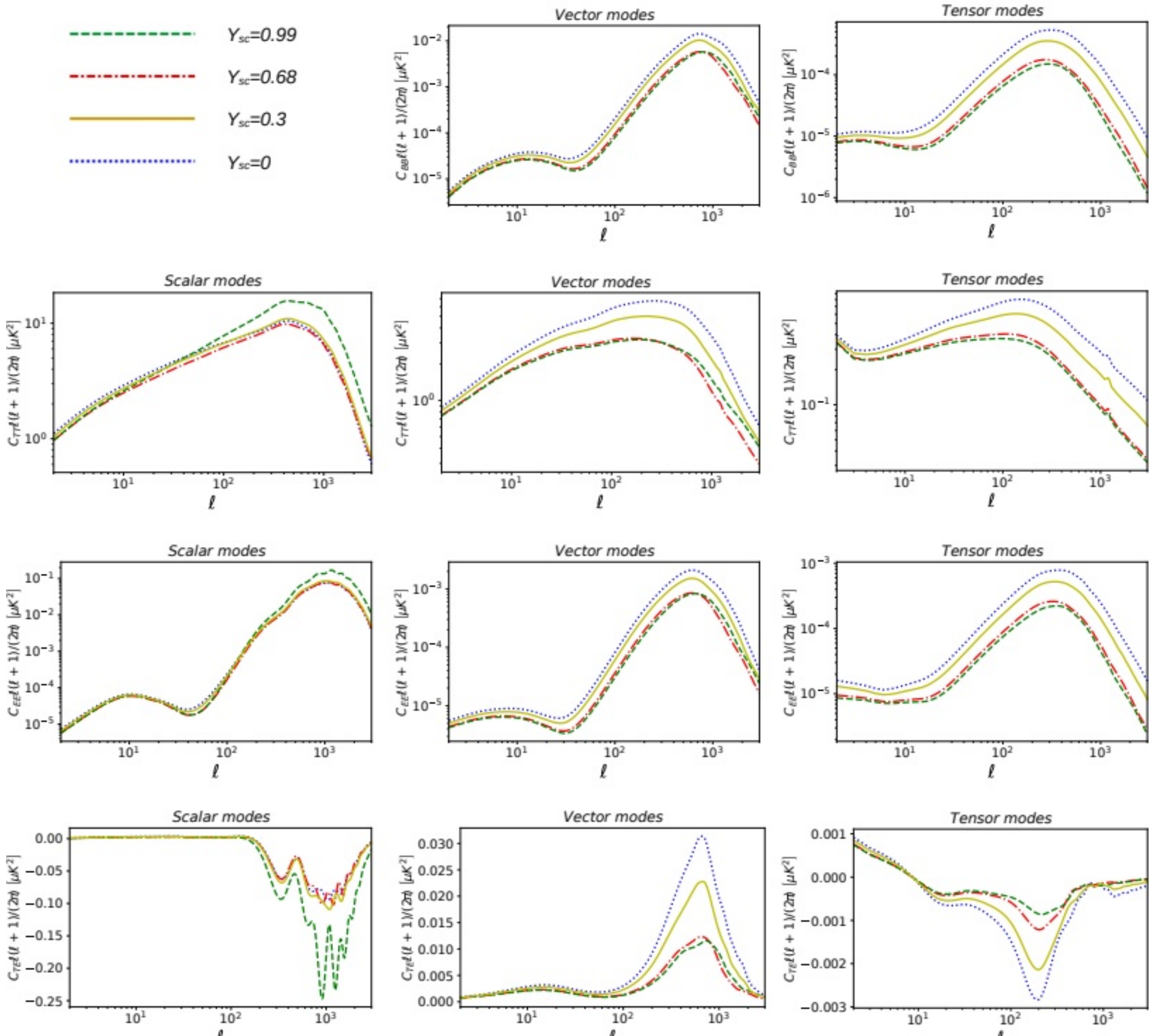
Fourier transform

$$\Theta^{\mu\nu} = \mu_0 \int_{-\xi_c/2}^{\xi_c/2} \left[(1+Y)\frac{\dot{X}^\mu \dot{X}^\nu}{\sqrt{1-v^2}} - (1-Y)\sqrt{1-v^2}X'^\mu X'^\nu - Y\left(\dot{X}^\mu X'^\nu + X'^\mu \dot{X}^\nu\right)\right] \mathrm{d}\sigma$$

→ SVT decomposition

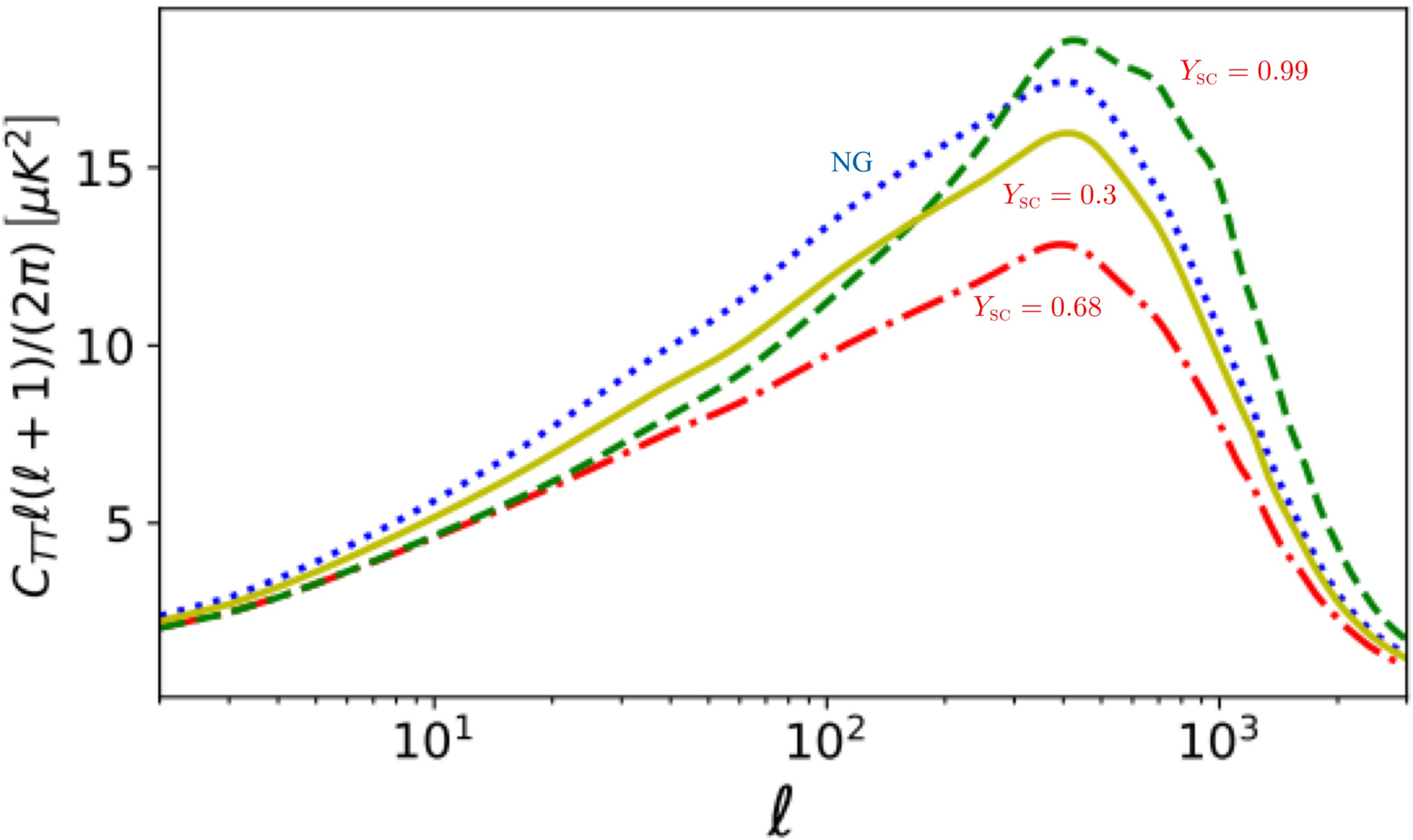
$$\Theta_{00} = \frac{\mu_0 \tilde{U}}{\sqrt{1-v^2}} \frac{\sin\left(\frac{1}{2}kX_3'\xi_c\right)}{\frac{1}{2}kX_3'} \cos\left(\boldsymbol{k}\cdot\boldsymbol{x}_0 + k\dot{X}_3 v\tau\right)$$

$$\begin{cases} \Theta^{\mathrm{S}} = \dfrac{1}{2}\left[v^2\left(3\dot{X}_3\dot{X}_3 - 1\right) - 6v\dfrac{Y}{1+Y}X_3'\dot{X}_3 - \left(1-v^2\right)\dfrac{1-Y}{1+Y}\left(3X_3'X_3' - 1\right)\right]\Theta_{00} \\ \Theta^{\mathrm{V}} = \left[v^2\dot{X}_1\dot{X}_3 - \dfrac{1-Y}{1+Y}(1-v^2)X_1'X_3' - v\dfrac{Y}{1+Y}\left(X_1'\dot{X}_3 + \dot{X}_1 X_3'\right)\right]\Theta_{00} \\ \Theta^{\mathrm{T}} = \left[v^2\dot{X}_1\dot{X}_2 - \dfrac{1-Y}{1+Y}(1-v^2)X_1'X_2' - v\dfrac{Y}{1+Y}\left(X_1'\dot{X}_2 + \dot{X}_1 X_2'\right)\right]\Theta_{00} \end{cases}$$

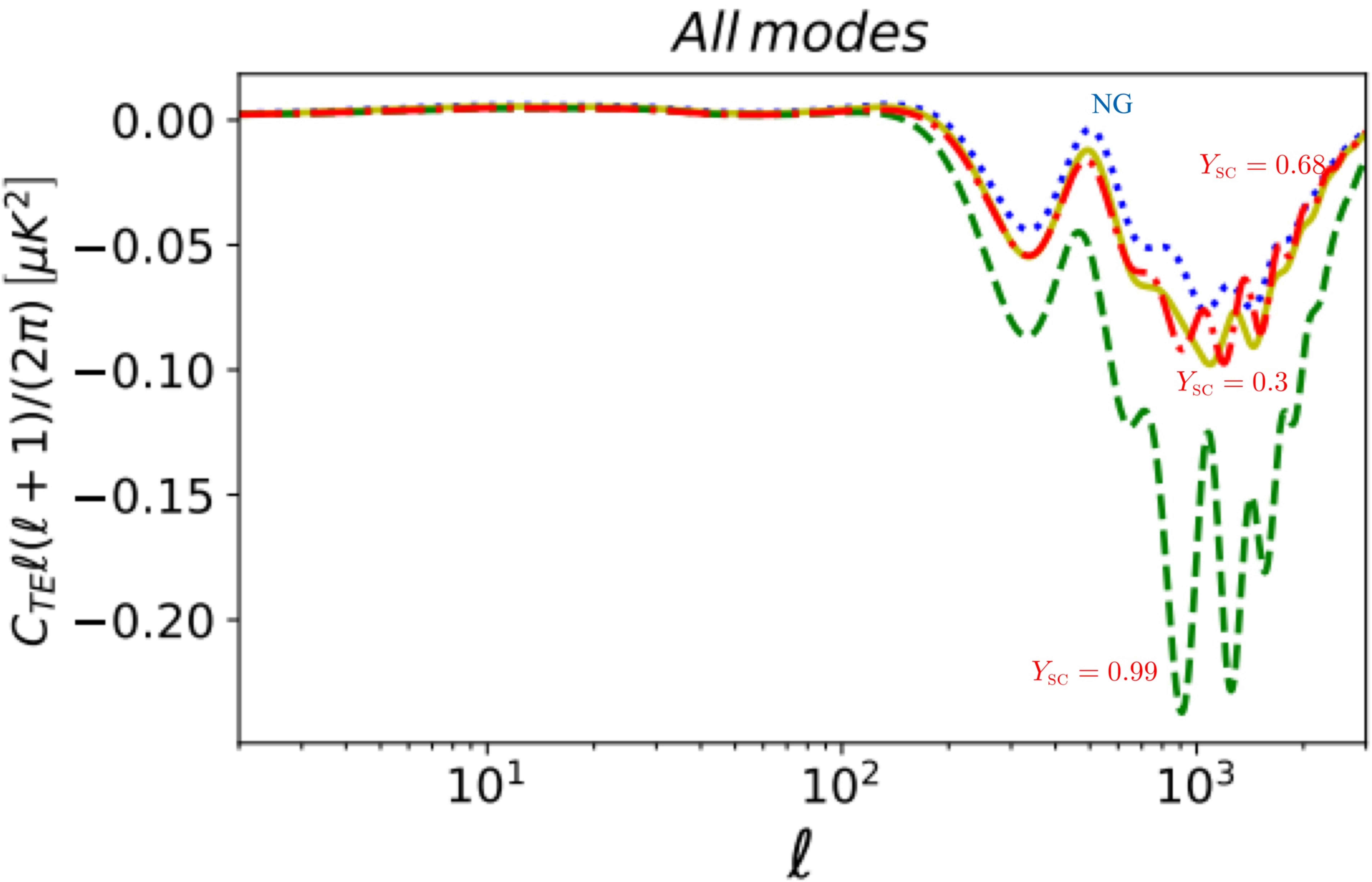


Ysc=0.99
Ysc=0.68
Ysc=0.3
Ysc=0
Scalar modes
Vector modes
Tensor modes
ℓ

All modes

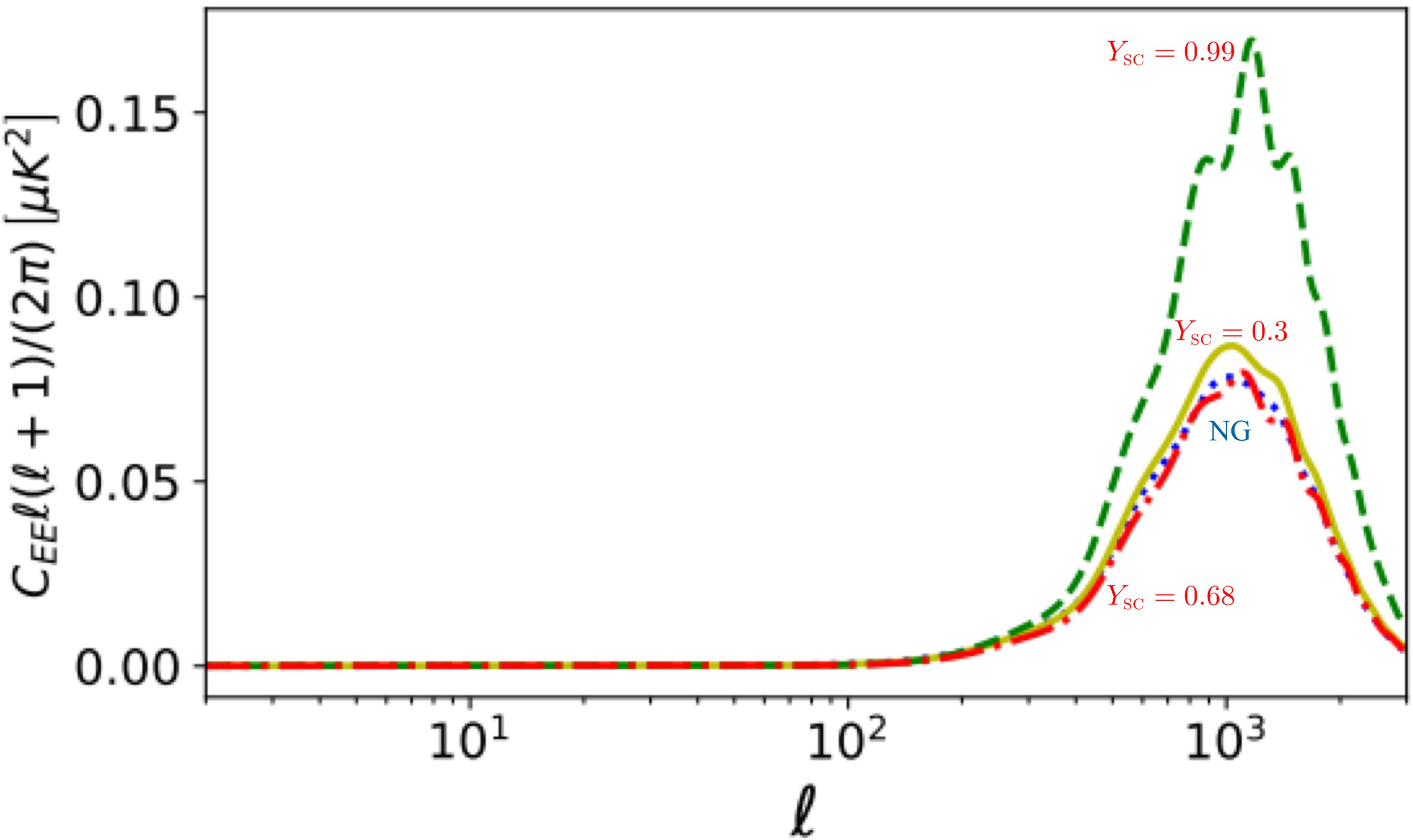


$C_{TT}\ell(\ell+1)/(2\pi)$ $[\mu K^2]$
15
10
5
10^1
10^2
10^3
ℓ
NG
$Y_{\rm sc} = 0.99$
$Y_{\rm sc} = 0.3$
$Y_{\rm sc} = 0.68$

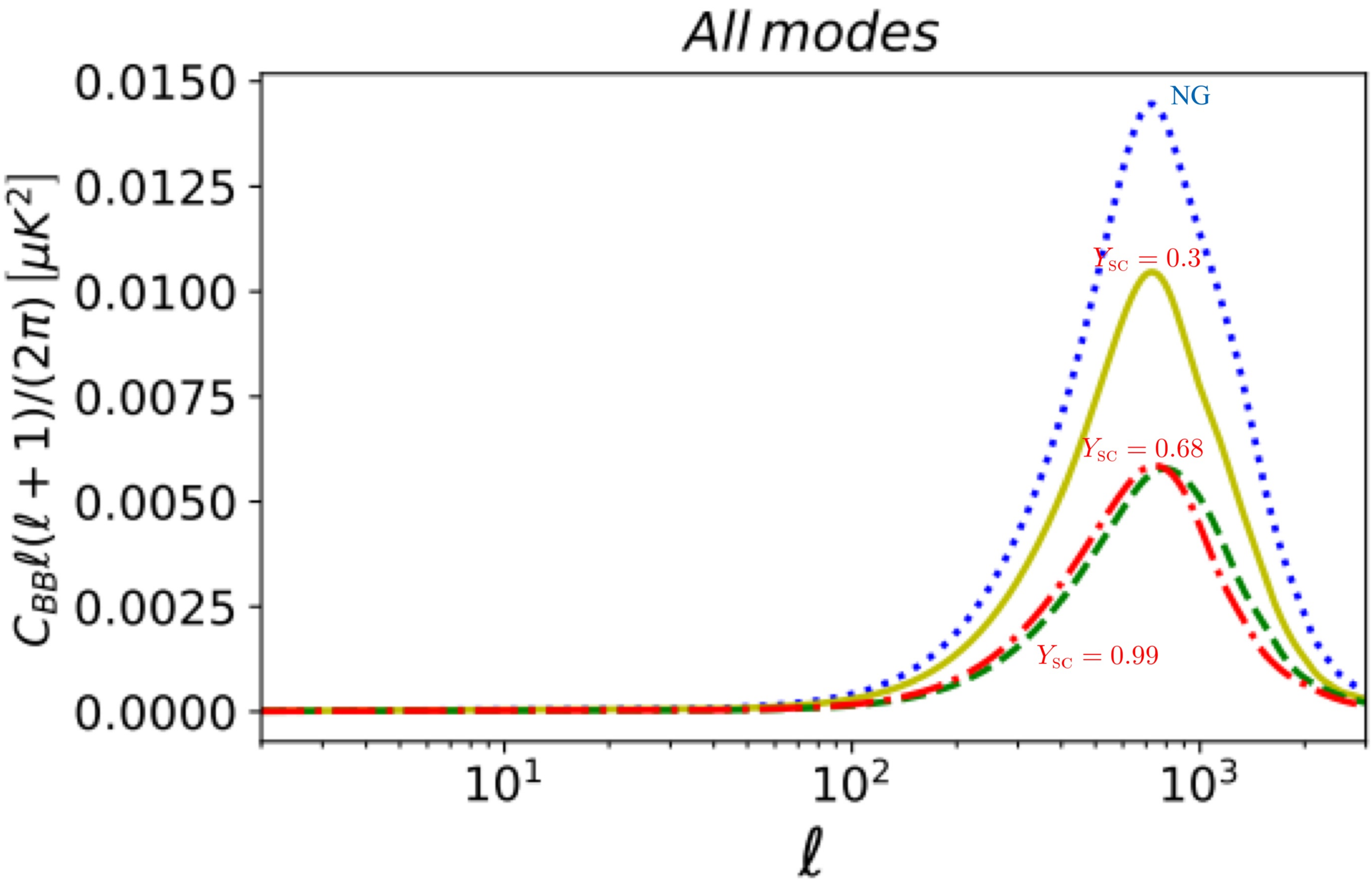


All modes
NG
$Y_{\rm SC} = 0.68$
$Y_{\rm SC} = 0.3$
$Y_{\rm SC} = 0.99$
0.00
−0.05
−0.10
−0.15
−0.20
$C_{TE}\ell(\ell+1)/(2\pi)$ $[\mu K^2]$
10^1
10^2
10^3
ℓ

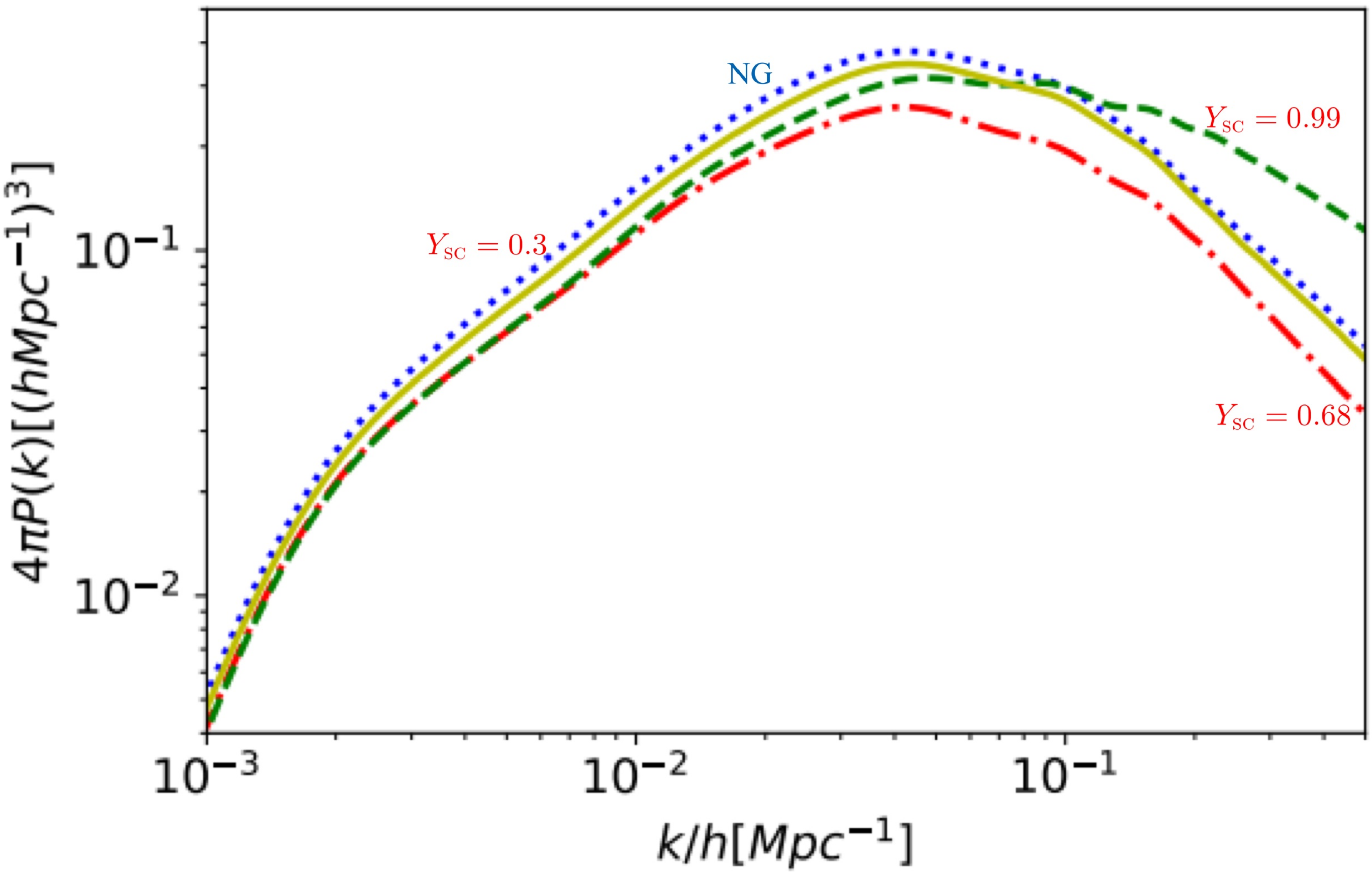
All modes

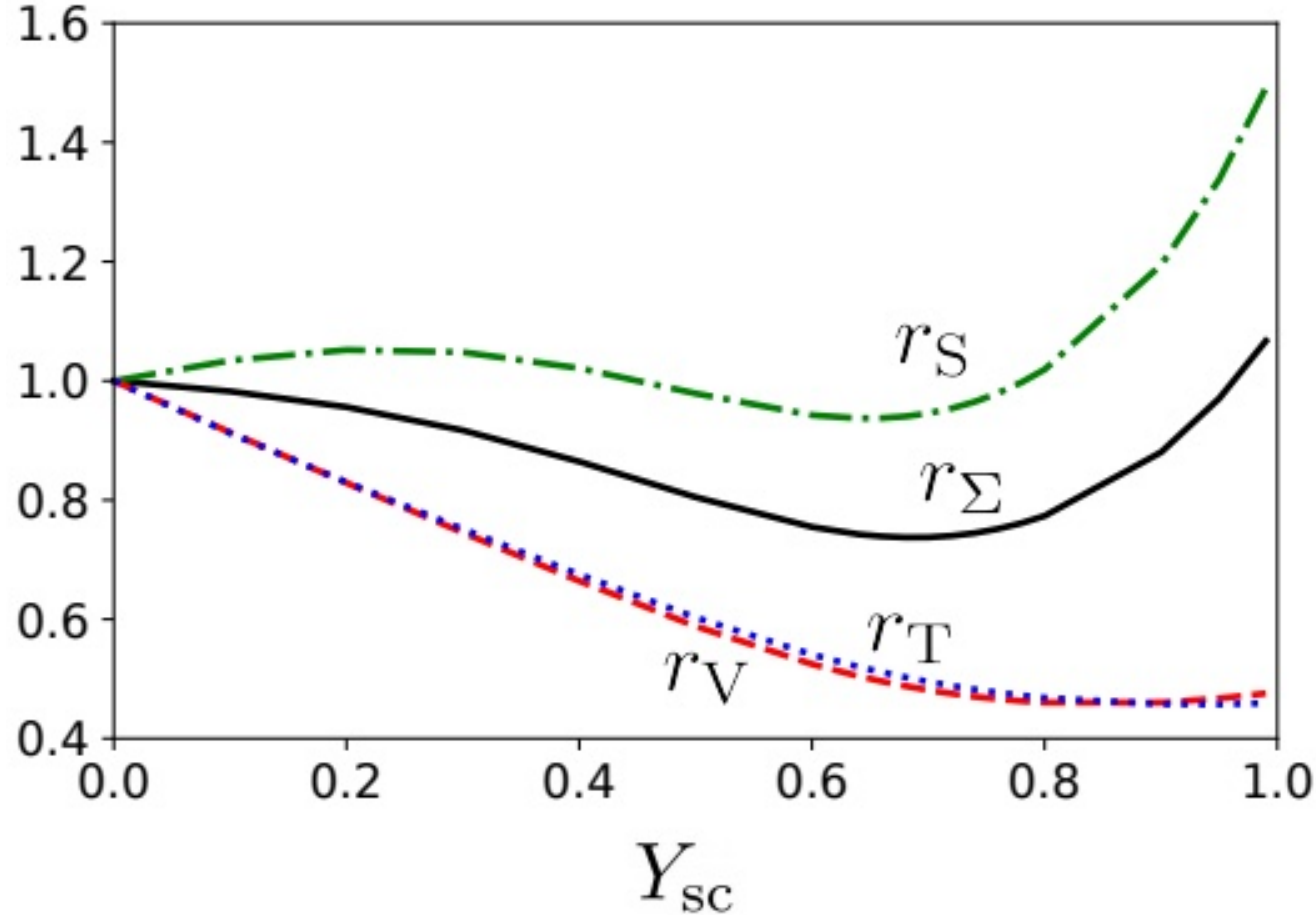


$Y_{\rm SC} = 0.99$
$Y_{\rm SC} = 0.3$
NG
$Y_{\rm SC} = 0.68$
$C_{EE}\ell(\ell+1)/(2\pi)\ [\mu K^2]$
0.15
0.10
0.05
0.00
10^1
10^2
10^3
ℓ


All modes
0.0150
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
$C_{BB}\ell(\ell+1)/(2\pi)\ [\mu K^2]$
NG
$Y_{\rm sc} = 0.3$
$Y_{\rm sc} = 0.68$
$Y_{\rm sc} = 0.99$
10^1
10^2
10^3
ℓ

Power spectrum




1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.0
0.2
0.4
0.6
0.8
1.0
r_S
r_Σ
r_T
r_V
Y_{sc}

Conclusions

- Current-carrying cosmic strings may be generic
- Thermodynamical parameters: velocity, correlation length, charge & current
- Many possible scenarios, one favoured ($Y_{\rm SC} \neq 0$ RDE & $Y_{\rm SC} = 0$ MDE)
- Can use CMBACT to investigate cosmologically observable consequences
- Possible amplitude reduction in C_ℓ of order $\sim 25\%$ and more power on large scales…

THANK YOU FOR YOUR ATTENTION