Asymptotic Accelerated Expansion in String Theory?

José Calderón Infante

Based on 2209.11821 with Ignacio Ruiz and Irene Valenzuela

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PART I

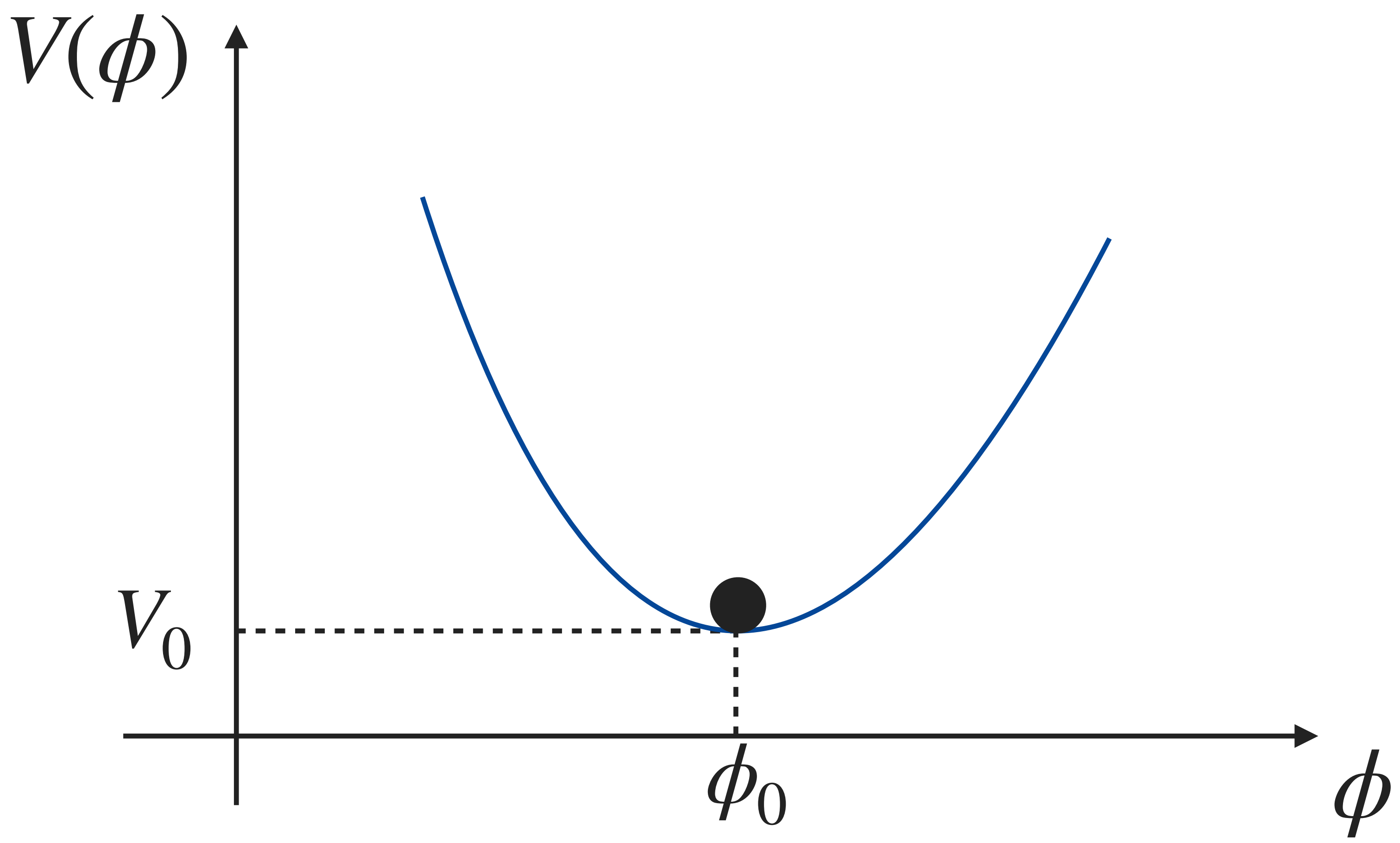
Introduction and Motivation

An Universe in Accelerated Expansion

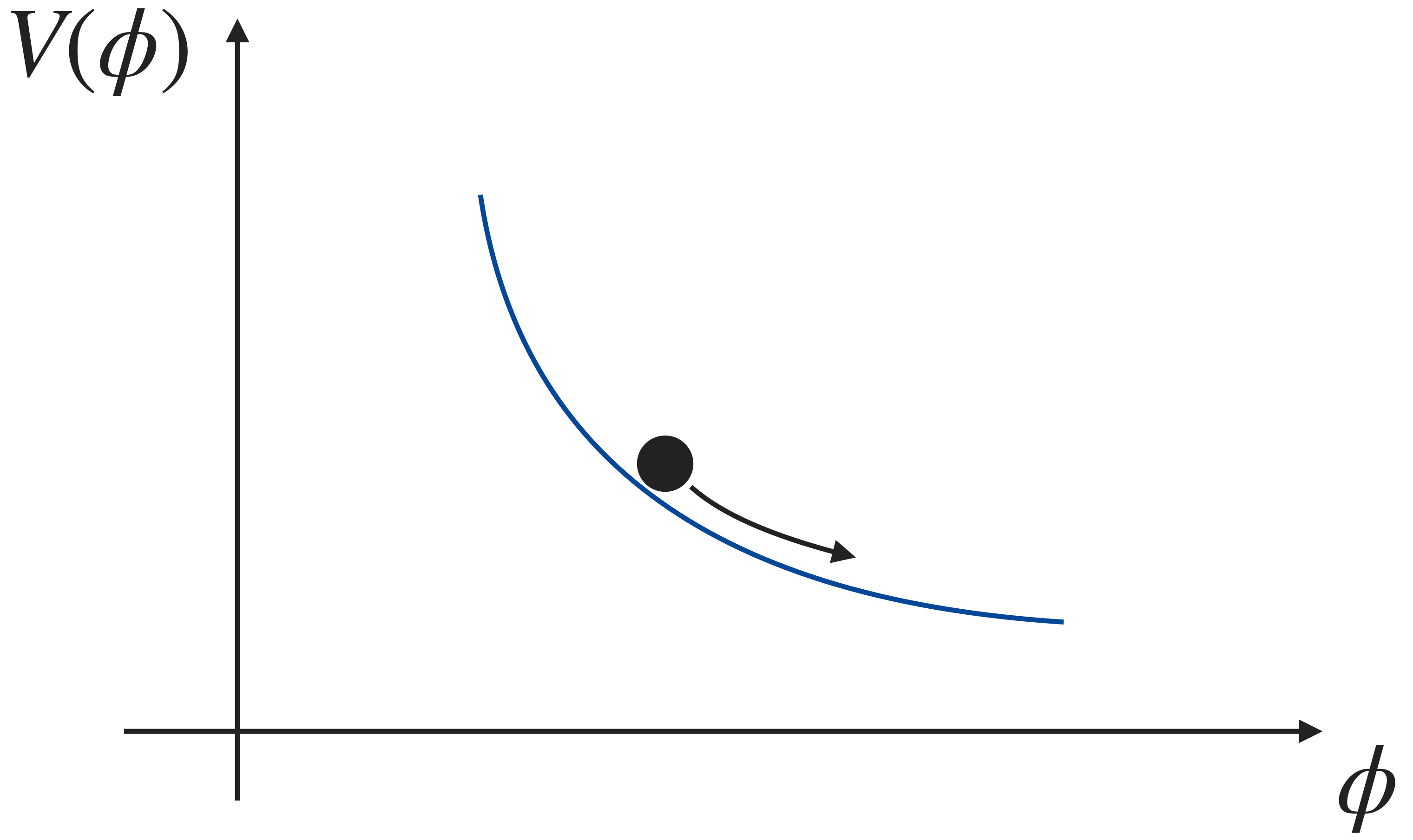
Our Universe is expanding in an accelerated fashion

How to describe this within EFT?

Dark Energy



Quintessence

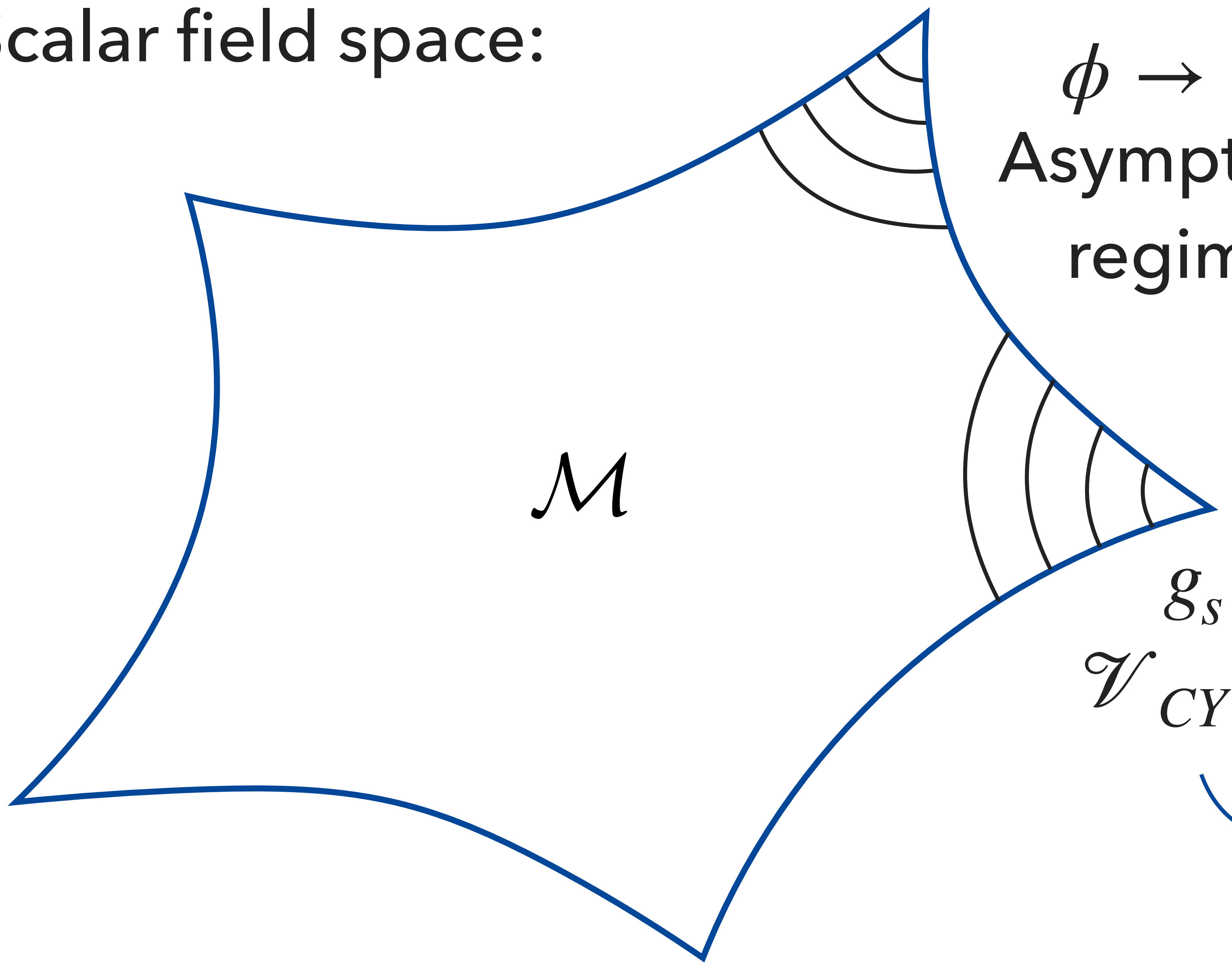


Important! We want this in an UV complete theory of Quantum Gravity

Try in String Theory!

Why Asymptotic?

Scalar field space:



$\phi \to \infty$
Asymptotic regime → Parametric control of the EFT description

Example:

$g_s \to 0$
$\mathcal{V}_{CY} \to \infty$ → String loop and alpha' corrections under parametric control

One example. Many more asymptotic limits in string theory

Runaway potentials at asymptotic limits → Quintessence going on forever

Will our Universe be in accelerated expansion forever?

Summary: Asymptotic accelerated expansion ⟶ Without quantum corrections and going on forever

The Swampland

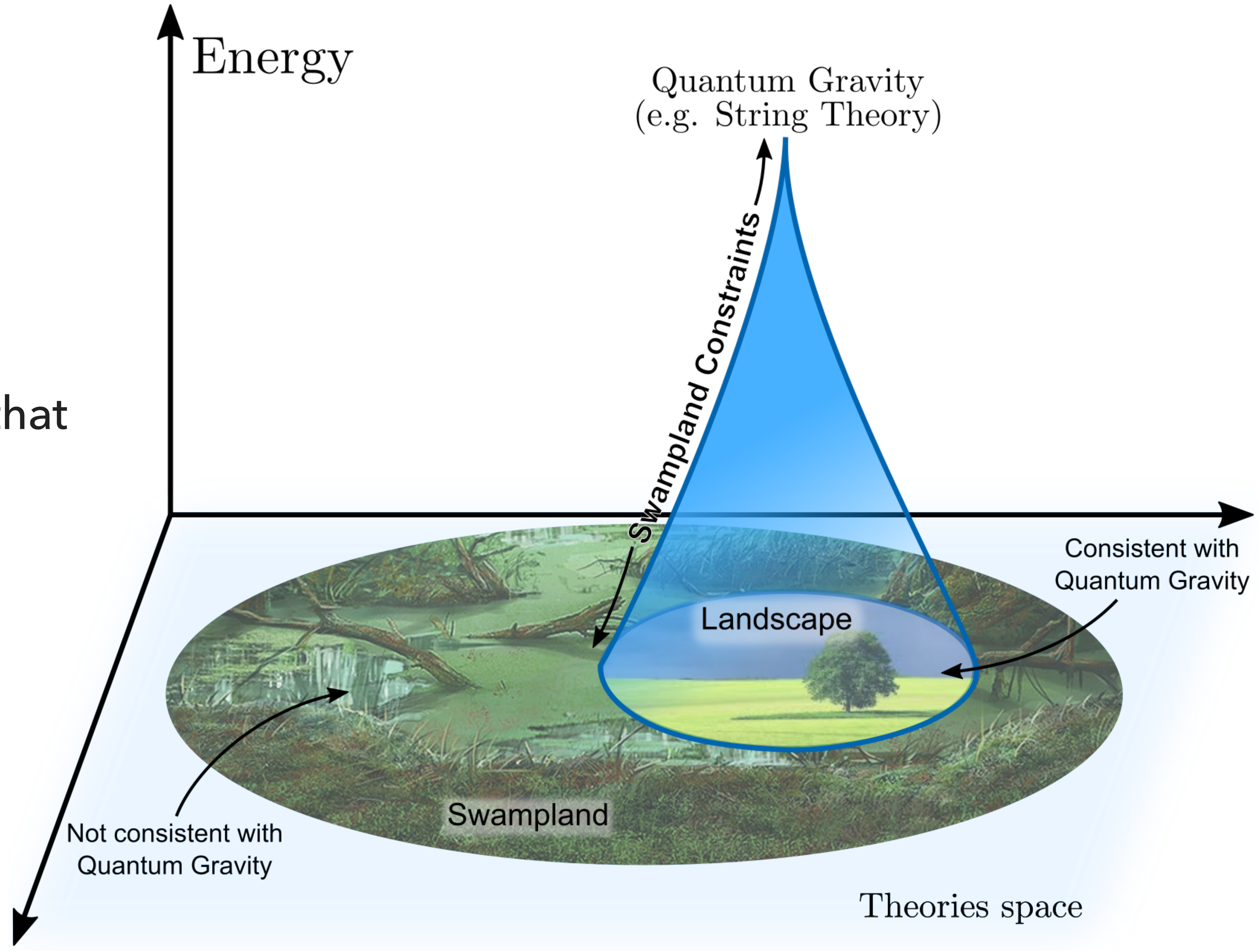
Is everything possible in Quantum Gravity ? → **No!**

Swampland: [Vafa '05]

Apparently consistent effective field theories that cannot be completed to quantum gravity

Swampland program:

Find constraints that effective field theories must satisfy so that they do not belong to the Swampland



[van Beest, JC, Mirfendereski, Valenzuela '21]

Asymptotic Accelerated Expansion and the Swampland

Swampland question: Is asymptotic accelerated expansion possible in Quantum Gravity?

Dark Energy

dS minima in String Theory? → **David's talk!**

➡ It has proven itself difficult to achieve!

No fully-stablished example + candidates need quantum corrections (e.g. [KKLT '03]) → No asymptotic regime!

Several no-go theorems valid in various asymptotic limits (e.g. [Grimm, Li, Valenzuela '19])

➡ Motivated some Swampland conjectures

Refined dS conjecture: Forbids dS minima! [Ooguri, Palti, Shiu, Vafa '18] [Garg, Krishnan '18]

Trans-Planckian Censorship Conjecture (TCC): dS vacua at best metastable! [Bedroya, Vafa '19]

→ Short-lived

Dark energy (if possible) expected to be $\begin{cases} \text{not under parametric control} \\ \text{not going on forever} \end{cases}$

Asymptotic Accelerated Expansion and the Swampland

Swampland question: Is asymptotic accelerated expansion possible in Quantum Gravity?

Quintessence

Asymptotic dS conjecture: $\gamma \equiv \frac{\nabla V(\phi)}{V(\phi)} \geq c_d \sim \mathcal{O}(1)$ as $\phi \to \infty$

Assuming gradient flow trajectories!

$V \sim e^{-\gamma\phi}$ as $\phi \to \infty$ ➡ Accelerated expansion $\longleftrightarrow \gamma < \frac{2}{\sqrt{d-2}}$

Asymptotic accelerated expansion? $\longrightarrow$ Depends on value of c_d

TCC: $c_{TCC} = \frac{2}{\sqrt{(d-1)(d-2)}}$ [Bedroya, Vafa '19]

Strong dS conjecture: $c_{strong} = \frac{2}{\sqrt{d-2}}$ [Rudelius '21]

No accelerated expansion !

But! Only tested at weak string coupling and large volume/complex-structure (e.g. [Cicoli, Cunillera, Padilla, Pedro '22]) ➡ **Our goal:** Consider more general asymptotic limits

PART II

Setup and Technical Tools

The Setup

F-theory on CY4 with fluxes $\longrightarrow$ $4d\ \mathcal{N}=1$ supergravity

$$S=\int d^4x\sqrt{-g}\left\{\frac{1}{2}R-\frac{1}{2}G_{I\bar{J}}\partial_\mu\Phi^I\partial^\mu\Phi^{\bar{J}}-V(\Phi,\bar{\Phi})\right\}$$

Field space metric

$\Phi^I=a^I+is^I$ $\boxed{\text{Complex structure}}$ + Kähler

[Grimm, Li, Valenzuela '18]

Asymptotic limits in complex structure $\longrightarrow$ Machinery of Mixed Hodge Theory

Discrete data charaterizing asymptotic limit

Two moduli limits: $s,u\to\infty$ $\Rightarrow$

$$\begin{cases} G_{I\bar{J}}d\Phi^I d\Phi^{\bar{J}}=\dfrac{\Delta d_s}{2s^2}\left(ds^2+da^2\right)+\dfrac{\Delta d_u}{2u^2}\left(du^2+db^2\right) \\ V\sim\displaystyle\sum_{(n,m)\in\mathcal{E}}\rho_{nm}{}^2 s^{n-4}u^{m-n} \end{cases}$$

[Grimm, Li, Valenzuela '19]

Axion polynomials: Axions and fluxes

Discrete numbers that depend on asymptotic limit

Turning off fluxes may take some terms to 0! $\Rightarrow$ $\boxed{\text{One asymptotic limit, many asymptotic potentials}}$

Gradient Flows and Geodesics

Multi-field cosmology $\longrightarrow$ What trajectory in field space?

Intuition! Asymptotically, trajectories should be gradient flows of the potential!

[Hetz, Palma '16] [Achúcarro, Palma '19]

Equations of motion: $\boxed{\text{Gradient flow} \longleftrightarrow \text{Geodesic}}$ Need to check!

e.g. [JC, Uranga, Valenzuela '20]

$$G_{I\bar{J}} d\Phi^I d\Phi^{\bar{J}} = \frac{\Delta d_s}{2s^2}\left(ds^2 + da^2\right) + \frac{\Delta d_u}{2u^2}\left(du^2 + db^2\right) \longrightarrow \text{Asymptotic geodesics: } a, b \to \text{const.}$$

✔

$$V \sim \sum_{(n,m)\in\mathscr{E}} \rho_{nm}{}^2 s^{n-4} u^{m-n}$$

Gradient flow such that $a, b \to$ const.

Acts as a mass term for axions!

$\Rightarrow$ $\boxed{\text{Gradient flows of the potential are good asymptotic trajectories for cosmology}}$

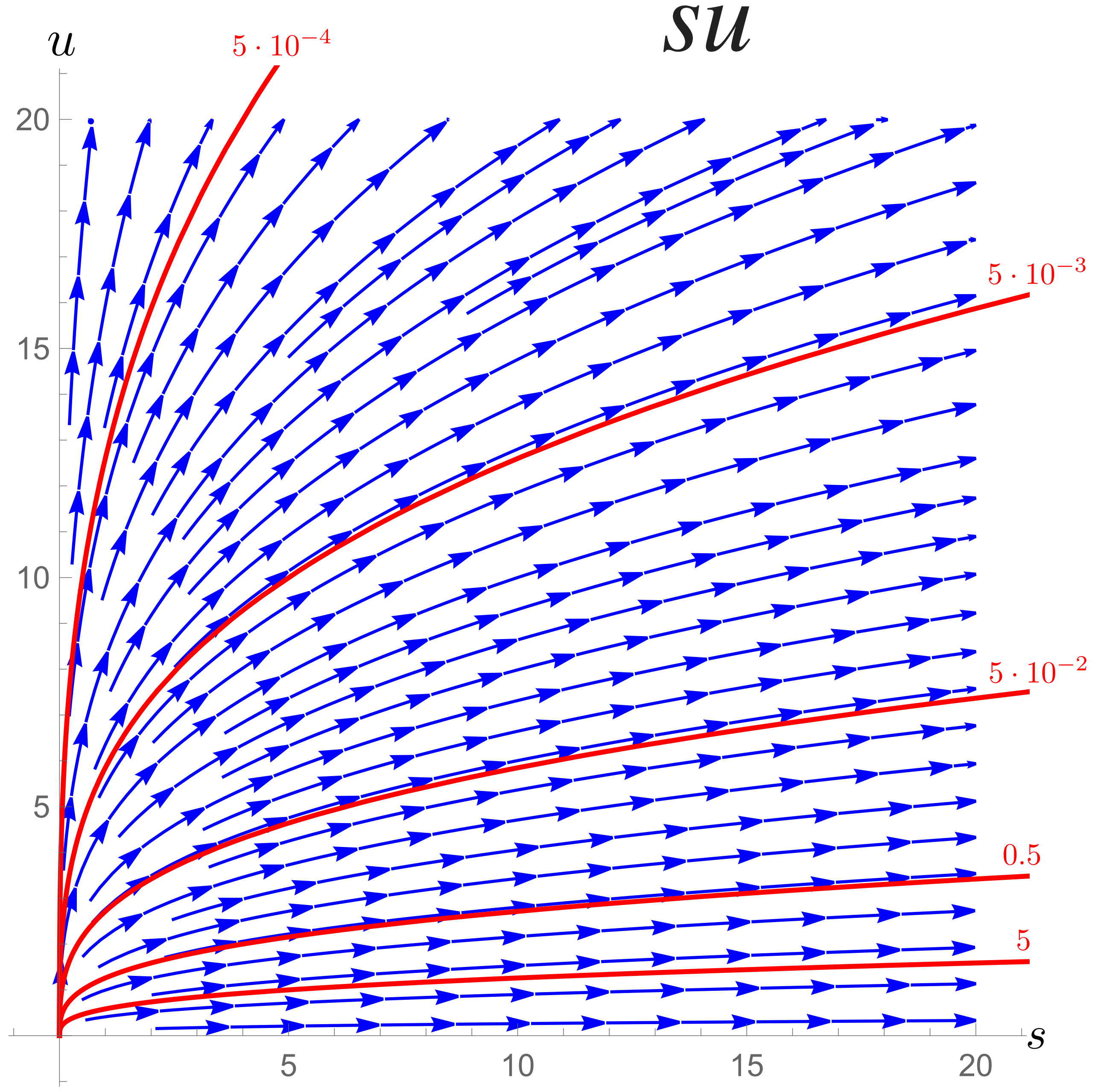
... the only ones at late times?

Saxion Gradient Flows: Two Scenarios

Scenario (I)

A single term dominates asymptotically

Example: $V = \dfrac{1}{su} + \cdots$

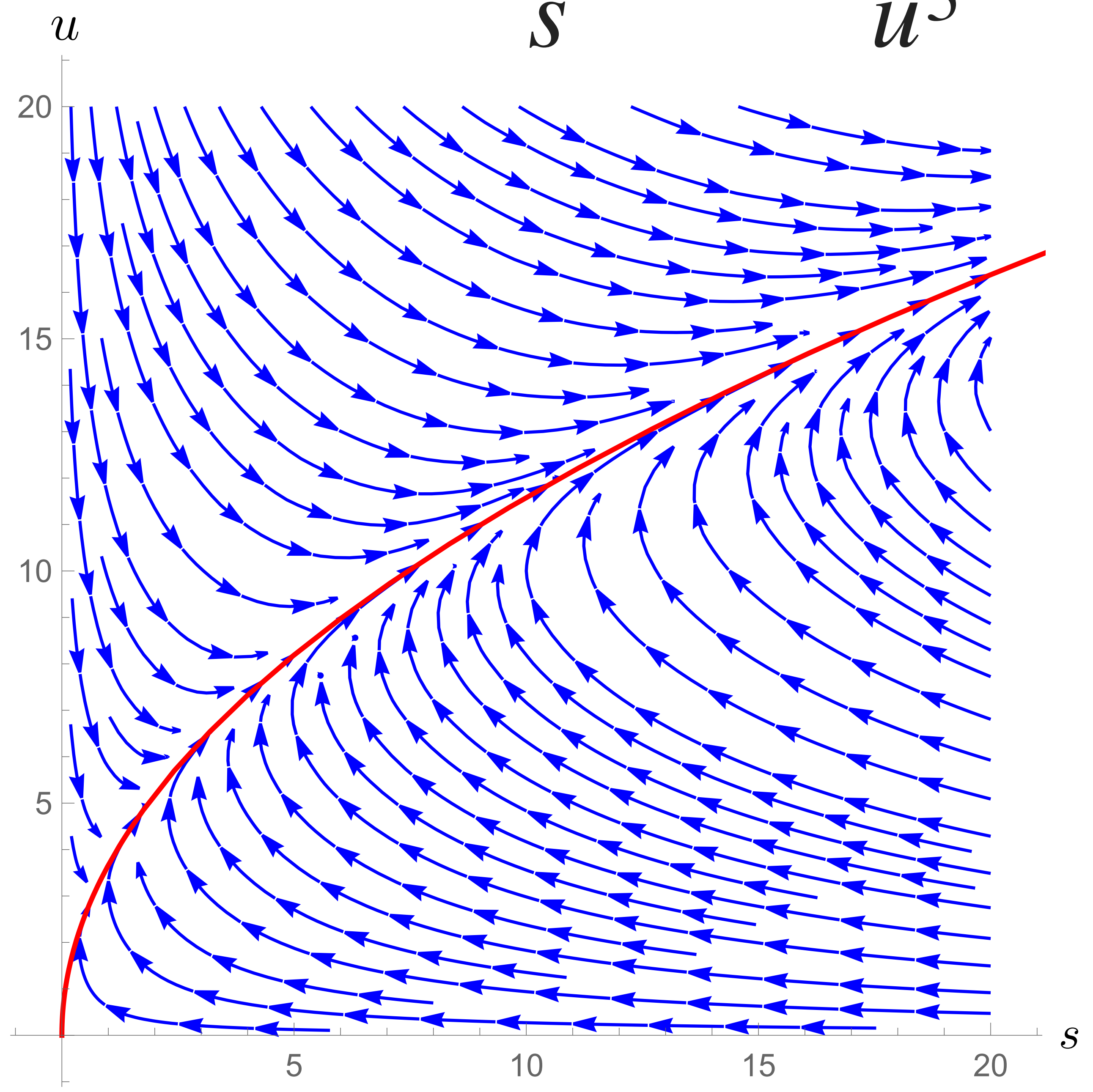


➡ Family of solutions $(s, u) = (\alpha\lambda^3, \lambda)$

NEW **Scenario (II)**

Several terms dominate asymptotically

Example: $V = \dfrac{u}{s} + 100\dfrac{s}{u^3} + \cdots$



➡ Unique solution $(s, u) = \left(\dfrac{\lambda^2}{10\sqrt{2}}, \lambda\right)$

Saxion Gradient Flows: Two Scenarios

Allows to violate previous bounds forbidding accelerated expansion!

- $4d\ \mathcal{N} = 1$ supergravity
  [Rudelius '21]

- Type IIB on CY3 orientifold with fluxes
  [Bastian, Grimm, van de Heisteeg '20]

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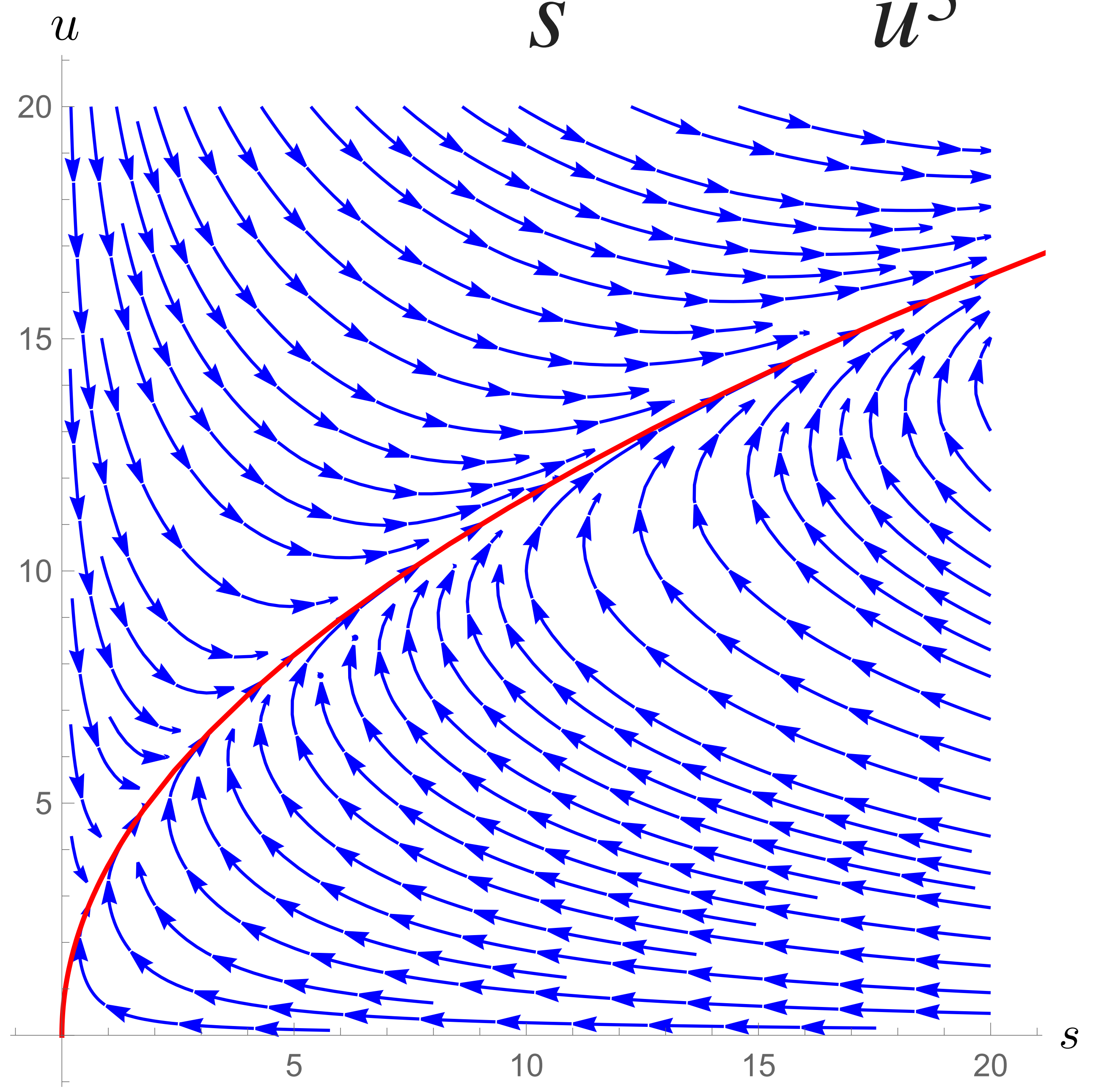
Example: $V = \dfrac{u}{s} + 100\dfrac{s}{u^3} + \cdots$



➡ Unique solution $(s, u) = \left(\dfrac{\lambda^2}{10\sqrt{2}}, \lambda\right)$

Convex Hull dS Conjecture

Geometric reformulation of asymptotic dS conjecture

Idea: Encode info about V and G_{ij} in some vectors

$$V = \sum_l V_l \longrightarrow \mu_l^a = -\,\delta^{ab}\, e_b^i \frac{\partial_i V_l}{V_l} \quad \textbf{dS ratios}$$

Orthonormal basis for G_{ij}

$\gamma =$ minimum distance to the convex hull of dS ratios !

Convex Hull dS Conjecture:
Convex hull of all dS ratios $\vec{\mu}_l$ must lie outside de ball or radius c_d

Similar to Convex Hull versions of

[Cheung, Remmen '14] Weak Gravity and Distance conjectures [JC, Uranga, Valenzuela '20]

[Arkani-Hamed, Motl, Nicolis, Vafa '07] [Ooguri, Vafa '07]

Convex Hull dS Conjecture

Geometric reformulation of asymptotic dS conjecture

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$$V = \sum_l V_l \longrightarrow \mu_l^a = -\delta^{ab} e_b^i \frac{\partial_i V_l}{V_l} \quad \textbf{dS ratios}$$

Orthonormal basis for G_{ij}

➡ $\gamma =$ minimum distance to the convex hull of dS ratios **!**

Convex Hull dS Conjecture:
Convex hull of all dS ratios $\vec{\mu}_l$ must lie outside de ball or radius c_d

Caveat: Relation to accelerated expansion restricted to gradient-flows=geodesics!

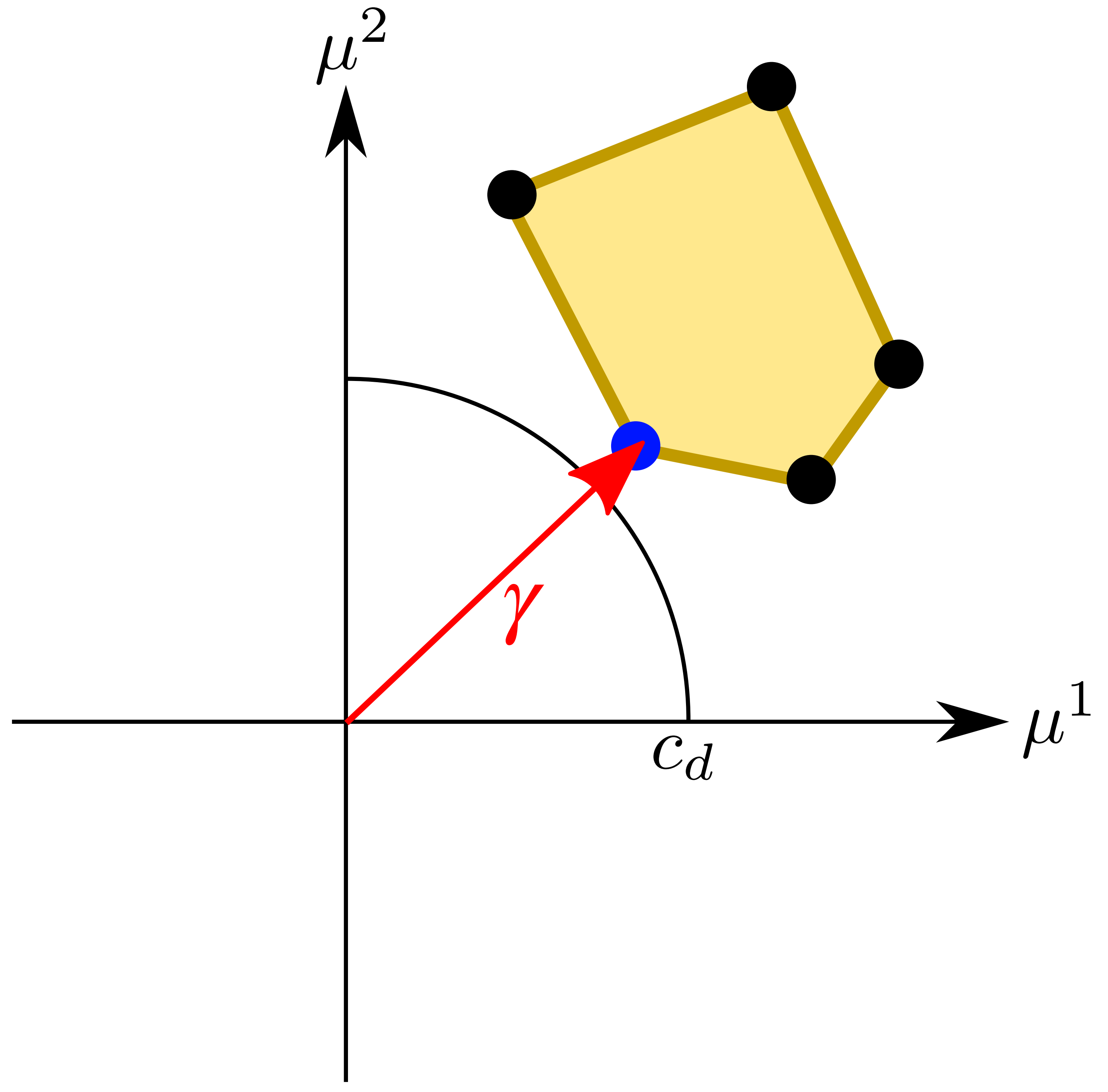
Until: [Shiu, Tonioni, Tran '23 (x2)] $\longrightarrow$ Convex hull condition works beyond gradient flows!

Reason: Non-gradient flows are less accelerated (in this setup) $\longrightarrow$ Highly non-trivial to show!

Convex Hull dS Conjecture

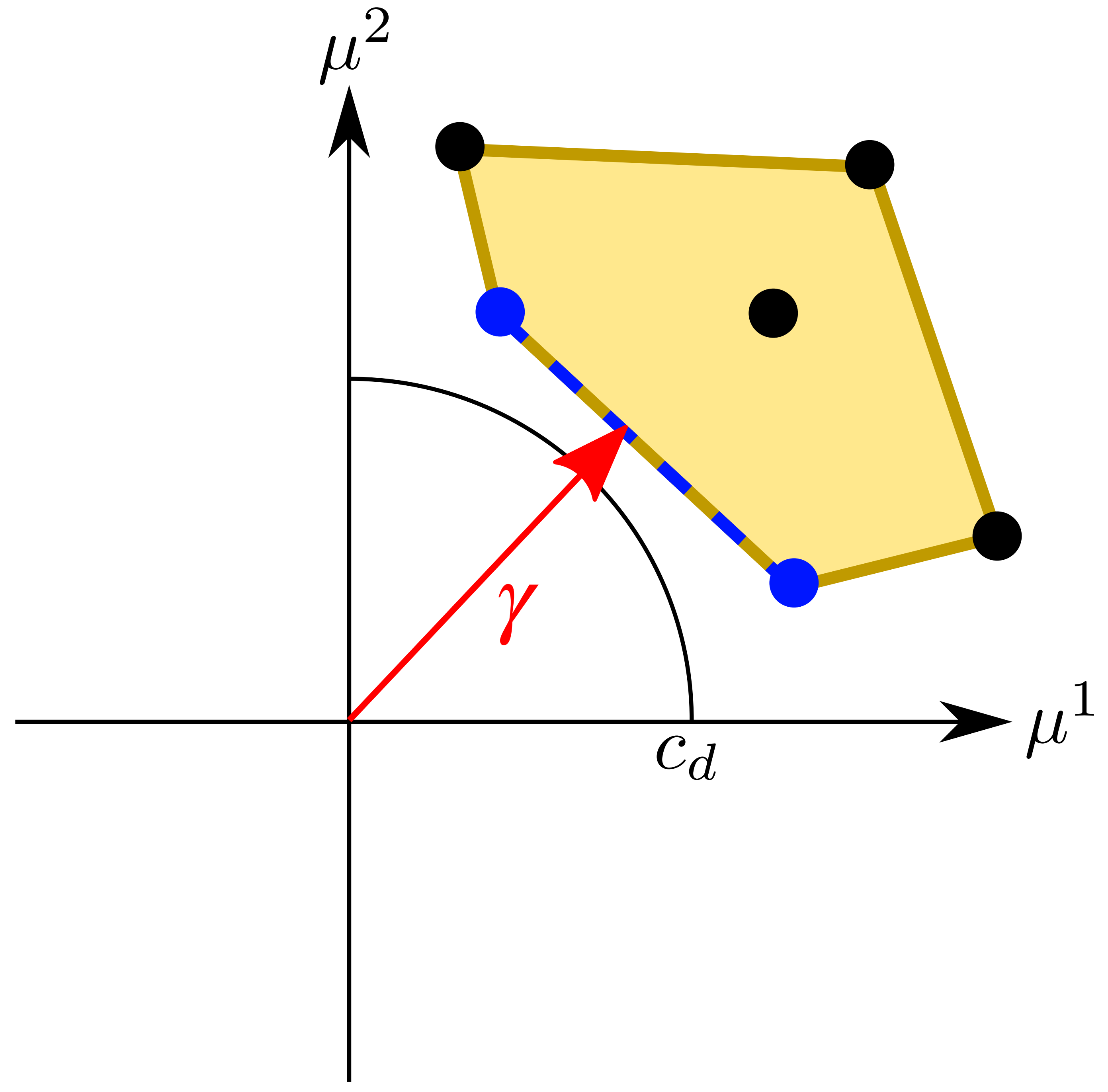
Scenario (I)

A single term dominates asymptotically



Scenario (II)

Several terms dominate asymptotically



Distance to the convex hull smaller than distance to each dS ratio!

Example: Type IIB Weak Coupling Large Complex Structure

In Mixed Hodge Theory language: $\mathrm{II}_{0,1} \to \mathrm{V}_{2,2}$ $\Rightarrow$ $\begin{cases} (\Delta d_s, \Delta d_u) = (1{,}3) \\ (n, m) = \{\dots\} \end{cases}$

$$V \sim \rho_{30}{}^2 \frac{1}{u^3 s} + \rho_{32}{}^2 \frac{1}{us} + \rho_{34}{}^2 \frac{u}{s} + \rho_{36}{}^2 \frac{u^3}{s} + \rho_{52}{}^2 \frac{s}{u^3} + \rho_{54}{}^2 \frac{s}{u} + \rho_{56}{}^2 us + \rho_{58}{}^2 u^3 s + \overbrace{\cancel{A_{44} - A_{\mathrm{loc}}}}^{\text{Irrelevant}}$$

Let us look at some of the axion polynomials

$$\rho_{52} = h_0 - h_1 b + \frac{1}{2} h_2 b^2 - \frac{1}{6} h_3 b^3$$

$-\partial_b$

$$\rho_{54} = h_1 - h_2 b + \frac{1}{2} h_3 b^2$$

$-\partial_b$

$$\rho_{56} = h_2 - h_3 b$$

$-\partial_b$

$$\rho_{58} = h_3$$

Example: Type IIB Weak Coupling Large Complex Structure

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$$\Downarrow$$

$$V \sim \left(\rho_{30}{}^{2}\frac{1}{u^3} + \rho_{32}{}^{2}\frac{1}{u} + \rho_{34}{}^{2}u + \rho_{36}{}^{2}u^3 \right)\frac{1}{s} + \left(\rho_{52}{}^{2}\frac{1}{u^3} + \rho_{54}{}^{2}\frac{1}{u} + \rho_{56}{}^{2}u + \rho_{58}{}^{2}u^3 \right) s + \overset{\text{Irrelevant}}{\cancel{A_{44} - A_{\mathrm{loc}}}}$$

$-\partial_b$ $\quad$ $-\partial_b$ $\quad$ $-\partial_b$

Each arrow leads to a more dominant term

… and to an axion polynomial of less degree!

Example: Type IIB Weak Coupling Large Complex Structure

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Encoded in an partially ordered chain!

$$\begin{tikzcd}
\rho_{30} \arrow[r] \arrow[d] & \rho_{32} \arrow[r] \arrow[d] & \rho_{34} \arrow[r] \arrow[d] & \rho_{36} \arrow[d] \\
\rho_{52} \arrow[r] & \rho_{54} \arrow[r] & \rho_{56} \arrow[r] & \rho_{58}
\end{tikzcd}$$

Leading term

$\rho_{58} \sim h_3{}^2$ Constant!

What if $h_3 = 0$?

Example: Type IIB Weak Coupling Large Complex Structure

In Mixed Hodge Theory language: $\mathrm{II}_{0,1} \rightarrow \mathrm{V}_{2,2}$ $\Rightarrow$ $\begin{cases} (\Delta d_s, \Delta d_u) = (1,3) \\ (n, m) = \{\dots\} \end{cases}$

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Encoded in an partially ordered chain !

$$\begin{tikzcd} \rho_{30} \arrow[r] \arrow[d] & \rho_{32} \arrow[r] \arrow[d] & \rho_{34} \arrow[r] \arrow[d] & \rho_{36} \sim \left(f_0 \pm \overset{0}{\cancel{h_3 c}}\right)^2 \\ \rho_{52} \arrow[r] & \rho_{54} \arrow[r] & \rho_{56} \sim \left(h_2 - \overset{0}{\cancel{h_3 b}}\right)^2 & \end{tikzcd}$$

Leading term

Leading term

Example: Type IIB Weak Coupling Large Complex Structure

In Mixed Hodge Theory language: $\mathrm{II}_{0,1} \rightarrow \mathrm{V}_{2,2}$ ➡ $\begin{cases} (\Delta d_s, \Delta d_u) = (1,3) \\ (n,m) = \{\ldots\} \end{cases}$

$$V \sim \rho_{30}{}^2 \frac{1}{u^3 s} + \rho_{32}{}^2 \frac{1}{us} + \rho_{34}{}^2 \frac{u}{s} + \rho_{36}{}^2 \frac{u^3}{s} + \rho_{52}{}^2 \frac{s}{u^3} + \rho_{54}{}^2 \frac{s}{u} + \rho_{56}{}^2 us + \cancel{\rho_{58}{}^2 u^3 s} + \cancel{A_{44} - A_{\mathrm{loc}}}$$

0 Irrelevant

Encoded in an partially ordered chain !

$$\begin{tikzcd} \rho_{30} \arrow[r] \arrow[d] & \rho_{32} \arrow[r] \arrow[d] & \rho_{34} \arrow[r] \arrow[d] & \rho_{36} \sim f_0{}^2 \\ \rho_{52} \arrow[r] & \rho_{54} \arrow[r] & \rho_{56} \sim h_2{}^2 & \end{tikzcd}$$

$\rho_{36} \sim f_0{}^2$ Constant!

Leading term

$\rho_{56} \sim h_2{}^2$ Constant!

Leading term

➡ Help us classify which terms can dominate and which are the relevant fluxes

Example: Type IIB Weak Coupling Large Complex Structure

In Mixed Hodge Theory language: $\mathrm{II}_{0,1} \rightarrow \mathrm{V}_{2,2}$ $\Rightarrow$ $\begin{cases} (\Delta d_s, \Delta d_u) = (1,3) \\ (n, m) = \{\dots\} \end{cases}$

Irrelevant

$$V \sim \rho_{30}{}^{2}\frac{1}{u^3 s} + \rho_{32}{}^{2}\frac{1}{us} + \rho_{34}{}^{2}\frac{u}{s} + \rho_{36}{}^{2}\frac{u^3}{s} + \rho_{52}{}^{2}\frac{s}{u^3} + \rho_{54}{}^{2}\frac{s}{u} + \overset{0}{\cancel{\rho_{56}{}^{2}us}} + \overset{0}{\cancel{\rho_{58}{}^{2}u^3 s}} + \cancel{A_{44} - A_{\mathrm{loc}}}$$

Encoded in an partially ordered chain !

$$\begin{tikzcd} \rho_{30} \arrow[r] \arrow[d] & \rho_{32} \arrow[r] \arrow[d] & \rho_{34} \arrow[r] & \rho_{36} \\ \rho_{52} \arrow[r] & \rho_{54} \end{tikzcd}$$

Gradient flow with $s, u \rightarrow \infty$ not possible if $V \rightarrow \infty$ $\Rightarrow$ Set $h_3 = h_2 = 0$

Example: Type IIB Weak Coupling Large Complex Structure

In Mixed Hodge Theory language: $\mathrm{II}_{0,1} \to \mathrm{V}_{2,2}$ $\Rightarrow$ $\begin{cases} (\Delta d_s, \Delta d_u) = (1,3) \\ (n,m) = \{\dots\} \end{cases}$

Potential	$\vec{\beta}$	$\gamma_{\vec{f}}$
$\frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{34}\frac{u}{s} + A_{36}\frac{u^3}{s} + A_{52}\frac{s}{u^3} + A_{54}\frac{s}{u} + A_{44} - A_{\rm loc}$	(-2,-1)	-
$\frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{34}\frac{u}{s} + A_{36}\frac{u^3}{s} + A_{52}\frac{s}{u^3} + A_{44} - A_{\rm loc}$	(3,1) $\star$	$2\sqrt{\frac{2}{3}}$
$\frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{52}\frac{s}{u^3} + A_{34}\frac{u}{s} + A_{54}\frac{s}{u} + A_{44} - A_{\rm loc}$	(1,1) $\star$	$\sqrt{2}$
$\frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{34}\frac{u}{s} + A_{36}\frac{u^3}{s}$	(1,0)	$\sqrt{2}$
$\frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{34}\frac{u}{s} + A_{52}\frac{s}{u^3}$	(2,1)	$\sqrt{\frac{2}{7}}$
$\frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{52}\frac{s}{u^3} + A_{54}\frac{s}{u}$	(-3,1)	-
$\frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{52}\frac{s}{u^3}$	(1,1)	$\sqrt{2}$
$\frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{34}\frac{u}{s}$	(1,0)	$\sqrt{2}$
$\frac{A_{30}}{su^3} + \frac{A_{32}}{us}$	(3,1)	$2\sqrt{\frac{2}{3}}$
$\frac{A_{30}}{su^3} + A_{52}\frac{s}{u^3}$	(-1,1)	-
$\frac{A_{30}}{su^3}$	(1,1)	$2\sqrt{2}$

Check
Table 2
of the paper !

PART III

Results and Discussion

Asymptotic Accelerated Expansion in String Theory?

We find two potential examples of asymptotic accelerated expansion!

- Type IIB at weak coupling and large complex structure:

$$V \sim f_2^2 \frac{u}{s} + h_0^2 \frac{s}{u^3} + \cdots \quad \Rightarrow \quad \gamma = \sqrt{\frac{2}{7}} < c_{TCC}$$

- A not weakly-coupled F-theory limit:

Both realize Scenario (II)

$$V \sim A_{42} \frac{1}{u^2} + A_{24} \frac{u^2}{s^2} + \cdots \quad \Rightarrow \quad \gamma = \frac{2}{\sqrt{5}} < c_{strong}$$

Caveat: Have to stabilize Kähler moduli, otherwise they contribute to γ

→ Required for phenomenologically viable quintessence model (5th forces)

→ Need to use quantum corrections! And probably metastable!

→ Caveat avoided in Type IIA… but we found no potential example there

Asymptotic Accelerated Expansion in String Theory?

Bonus track: Recent exciting results!

[Cremonini, Gonzalo, Rajaguru, Tang, Wrase '23]

No asymptotic accelerated expansion in a model without Kähler moduli

[Hebecker, Schreyen, Venken '23]

Suggest: Asymptotic accelerated expansion ↔ dS in more dimensions!

➡ If one is hard, the other too!

[Andriot, Tsimpis, Wrase '23]

Asymptotic accelerated expansion for open universes in String Theory

… without cosmological horizon!

➡ Asymptotic accelerated expansion ok, but QG abhors cosmological horizons ?

Future Directions and Conclusions

Future directions

Casuistics

More moduli, more general limits

More model independent approach

Beyond asymptotic regime: Quantum corrections, Kähler stabilization

→ Technically challenging, and thus exciting !

My conclusions/feeling

- Accelerated expansion in String Theory will require knowledge about quantum corrections

- Accelerated expansion in quantum gravity cannot last forever

Absence of asymptotic observables, holography? [Rudelius '21] [Bedroya, 23]

QG abhors cosmological horizons? [Andriot, Tsimpis, Wrase '23]

→ Prediction for the future or our own Universe !

Thank you for your attention!