Asymptotic Accelerated Expansion in String Theory?

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PART I

Introduction and Motivation
An Universe in Accelerated Expansion

Our Universe is expanding in an accelerated fashion

How to describe this within EFT?

Important! We want this in an UV complete theory of Quantum Gravity

Try in String Theory!
Why Asymptotic?

Scalar field space:

\[ \phi \rightarrow \infty \]

Asymptotic regime

Parametric control of the EFT description

Example:

\[ g_s \rightarrow 0 \]

\[ V_{CY} \rightarrow \infty \]

String loop and alpha’ corrections under parametric control

One example. Many more asymptotic limits in string theory

Runaway potentials at asymptotic limits

Quintessence going on forever

Will our Universe be in accelerated expansion forever?

Summary: Asymptotic accelerated expansion \( \rightarrow \) Without quantum corrections and going on forever
Swampland: [Vafa ’05]

**Swampland** is the term used to describe apparently consistent effective field theories that cannot be completed to quantum gravity. These theories are consistent with quantum gravity and are part of a larger landscape of possible theories. However, not all theories in this landscape are consistent with quantum gravity, and some theories are not consistent with each other.

**Swampland program:**

Find constraints that effective field theories must satisfy so that they do not belong to the Swampland.

Is everything possible in Quantum Gravity?

No!
**Swampland question:** Is asymptotic accelerated expansion possible in Quantum Gravity?

**Dark Energy**
- dS minima in String Theory?
- It has proven itself difficult to achieve!
- No asymptotic regime!
- No fully-stabilized example + candidates need quantum corrections (e.g. [KKLT '03])
- Several no-go theorems valid in **various asymptotic limits** (e.g. [Grimm, Li, Valenzuela '19])

**Refined dS conjecture:** Forbids dS minima! [Ooguri, Palti, Shiu, Vafa '18] [Garg, Krishnan '18]

**Trans-Planckian Censorship Conjecture (TCC):** dS vacua at best metastable! [Bedroya, Vafa '19]

**Motivated some Swampland conjectures**
- Dark energy (if possible) expected to be **not under parametric control**
- **not going on forever**

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**David's talk!**
Swampland question: Is asymptotic accelerated expansion possible in Quantum Gravity?

Quintessence

Asymptotic dS conjecture: $\gamma \equiv \frac{\nabla V(\phi)}{V(\phi)} \geq c_d \sim \Theta(1)$ as $\phi \to \infty$

$V \sim e^{-\gamma \phi}$ as $\phi \to \infty$ → Accelerated expansion $\gamma < \frac{2}{\sqrt{d-2}}$

Asymptotic accelerated expansion? → Depends on value of $c_d$

TCC: $c_{TCC} = \frac{2}{\sqrt{(d-1)(d-2)}}$ [Bedroya, Vafa ‘19]

Strong dS conjecture: $c_{strong} = \frac{2}{\sqrt{d-2}}$ [Rudelius ‘21] → No accelerated expansion!

But! Only tested at weak string coupling and large volume/complex-structure (e.g. [Cicoli, Cunillera, Padilla, Pedro ‘22])

Our goal: Consider more general asymptotic limits
The Setup

F-theory on CY4 with fluxes $\rightarrow$ 4d $\mathcal{N} = 1$ supergravity

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} R - \frac{1}{2} G_{IJ} \partial^\mu \Phi^I \partial_\mu \Phi^J - V(\Phi, \bar{\Phi}) \right\}$$

Field space metric $\Phi^I = a^I + i s^I$ Complex structure + Kähler

[Grimm, Li, Valenzuela ‘18]

Asymptotic limits in complex structure $\rightarrow$ Machinery of Mixed Hodge Theory

Discrete data characterizing asymptotic limit

$$G_{IJ} d\Phi^I d\Phi^J = \frac{\Delta d_s}{2 s^2} (ds^2 + da^2) + \frac{\Delta d_u}{2 u^2} (du^2 + db^2)$$

$$V \sim \sum_{(n,m) \in \mathcal{E}} \rho_{nm} 2^{s-n-4} u^{m-n}$$

[Grimm, Li, Valenzuela ‘19]

Discrete numbers that depend on asymptotic limit

Axion polynomials: Axions and fluxes

Turning off fluxes may take some terms to 0!

One asymptotic limit, many asymptotic potentials
Gradient Flows and Geodesics

Multi-field cosmology → What trajectory in field space?

**Intuition!** Asymptotically, trajectories should be gradient flows of the potential!

\[ G_{I\overline{J}} d\Phi^I d\Phi^{\overline{J}} = \frac{\Delta s^2}{2s^2} \left( ds^2 + da^2 \right) + \frac{\Delta u^2}{2u^2} \left( du^2 + db^2 \right) \]

\[ V \sim \sum_{(n,m) \in \mathcal{E}} \rho_{nm} \frac{s^{n-4}}{u^{m-n}} \]

Acts as a mass term for axions!

Equations of motion: [Gradient flow ↔ Geodesic] Need to check!

Asymptotic geodesics: \( a, b \to \text{const.} \)

Gradient flow such that \( a, b \to \text{const.} \)

\[ \text{Gradient flows of the potential are good asymptotic trajectories for cosmology} \]

... the only ones at late times?
Saxion Gradient Flows: Two Scenarios

Scenario (I)
A single term dominates asymptotically

Example: \( V = \frac{1}{s u} + \cdots \)

Family of solutions \((s, u) = (\alpha \lambda^3, \lambda)\)

NEW

Scenario (II)
Several terms dominate asymptotically

Example: \( V = \frac{u}{s} + 100\frac{s}{u^3} + \cdots \)

Unique solution \((s, u) = \left(\frac{\lambda^2}{10\sqrt{2}}, \lambda\right)\)
Saxion Gradient Flows: Two Scenarios

Allows to violate previous bounds forbidding accelerated expansion!

- $4d \mathcal{N} = 1$ supergravity
  [Rudelius ‘21]

- Type IIB on CY3 orientifold with fluxes
  [Bastian, Grimm, van de Heisteeg ‘20]

Several terms dominate asymptotically

$V = \frac{u}{s} + 100\frac{s}{u^3} + \cdots$

Example:

Unique solution $(s, u) = \left( \frac{\lambda^2}{10\sqrt{2}}, \lambda \right)$
Convex Hull dS Conjecture

Geometric reformulation of asymptotic dS conjecture

**Idea:** Encode info about $V$ and $G_{ij}$ in some vectors

$$V = \sum_l V_l \quad \rightarrow \quad \mu^a_l = - \delta^{ab} e^i_b \frac{\partial_i V_l}{V_l}$$

**dS ratios**

Orthonormal basis for $G_{ij}$

$$\gamma = \text{minimum distance to the convex hull of dS ratios}$$

**Convex Hull dS Conjecture:**
Convex hull of all dS ratios $\vec{\mu}_l$ must lie **outside** de ball or radius $c_d$

Similar to Convex Hull versions of

[Cheung, Remmen ’14] **Weak Gravity and Distance conjectures** [JC, Uranga, Valenzuela ’20]

[Arkani-Hamed, Motl, Nicolis, Vafa ’07] [Ooguri, Vafa ’07]
Convex Hull dS Conjecture

Geometric reformulation of asymptotic dS conjecture

**Idea:** Encode info about $V$ and $G_{ij}$ in some vectors

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$dS$ ratios

Orthonormal basis for $G_{ij}$

$\gamma = \text{minimum distance to the convex hull of } dS \text{ ratios}$

**Convex Hull dS Conjecture:**
Convex hull of all dS ratios $\vec{\mu}_l$ must lie outside de ball or radius $c_d$

**Caveat:** Relation to accelerated expansion restricted to gradient-flows=geodesics!

**Until:** [Shiu, Tonioni, Tran ‘23 (x2)] $\rightarrow$ Convex hull condition works beyond gradient flows!

**Reason:** Non-gradient flows are less accelerated (in this setup) $\rightarrow$ Highly non-trivial to show!
Convex Hull dS Conjecture

**Scenario (I)**
A single term dominates asymptotically

**Scenario (II)**
Several terms dominate asymptotically

Distance to the convex hull smaller than distance to each dS ratio!
Example: Type IIB Weak Coupling Large Complex Structure

In Mixed Hodge Theory language: \( \Pi_{0,1} \rightarrow V_{2,2} \)

\[
V \sim \rho_{30} \frac{2}{u^3s} + \rho_{32} \frac{2}{us} + \rho_{34} \frac{2u}{s} + \rho_{36} \frac{2u^3}{s} + \rho_{52} \frac{2s}{u^3} + \rho_{54} \frac{2s}{u} + \rho_{56} \frac{2us}{u} + \rho_{58} \frac{2u^3s}{u} + A_{44} - A_{loc}
\]

Let us look at some of the axion polynomials

\[
\rho_{52} = h_0 - h_1b + \frac{1}{2}h_2b^2 - \frac{1}{6}h_3b^3 - \partial_b
\]

\[
\rho_{54} = h_1 - h_2b + \frac{1}{2}h_3b^2 - \partial_b
\]

\[
\rho_{56} = h_2 - h_3b - \partial_b
\]

\[
\rho_{58} = h_3 - \partial_b
\]
In Mixed Hodge Theory language: $\Pi_{0,1} \rightarrow V_{2,2}$

\[
V \sim \rho_{30} \frac{2}{u^3 s} + \rho_{32} \frac{2}{u s} + \rho_{34} \frac{2}{s} + \rho_{36} \frac{2}{u^3} + \rho_{52} \frac{2}{s} + \rho_{54} \frac{2}{u} + \rho_{56} \frac{2}{u s} + \rho_{58} \frac{2}{u^3 s} + A_{44} - A_{\text{loc}}
\]

… and to an axion polynomial of less degree!
Example: Type IIB Weak Coupling Large Complex Structure

In Mixed Hodge Theory language: \( I_{0,1} \rightarrow V_{2,2} \)

\[
(V \sim \rho_{30} \frac{2}{u^3 s} + \rho_{32} \frac{2}{u s} + \rho_{34} \frac{2}{u^3} + \rho_{36} \frac{2}{u^3 s} + \rho_{52} \frac{2}{u} + \rho_{54} \frac{2}{u^3} + \rho_{56} \frac{2}{u s} + \rho_{58} \frac{2}{u^3 s} + A_{44} - A_{loc})
\]

Encoded in an partially ordered chain

\( \rho_{30} \rightarrow \rho_{32} \rightarrow \rho_{34} \rightarrow \rho_{36} \)

\( \rho_{52} \rightarrow \rho_{54} \rightarrow \rho_{56} \rightarrow \rho_{58} \)

Leading term

\( \rho_{58} \sim h_3^2 \) Constant!

What if \( h_3 = 0 \)?
Example: Type IIB Weak Coupling Large Complex Structure

In Mixed Hodge Theory language: $\Pi_{0,1} \rightarrow V_{2,2}$

$$\left\{ \begin{align*}
(\Delta d_s, \Delta d_u) &= (1,3) \\
(n, m) &= \{\ldots\}
\end{align*} \right.$$ 

$$V \sim \rho_{30} \frac{2}{u^3 s} + \rho_{32} \frac{2}{u s} + \rho_{34} \frac{2 u}{s} + \rho_{36} \frac{2 u^3}{s} + \rho_{52} \frac{2 s}{u^3} + \rho_{54} \frac{2 s}{u} + \rho_{56} \frac{2 u s}{u^3 s} + \rho_{58} \frac{2 u^3 s}{u^3 s} + A_{44} - A_{loc}$$

Encoded in a partially ordered chain:

- $\rho_{30} \rightarrow \rho_{32} \rightarrow \rho_{34} \rightarrow \rho_{36} \sim (f_0 \pm h_3 c)^2$
- $\rho_{52} \rightarrow \rho_{54} \rightarrow \rho_{56} \sim (h_2 - h_3 d)^2$

Leading term
Example: Type IIB Weak Coupling Large Complex Structure

In Mixed Hodge Theory language: \( \Pi_{0,1} \rightarrow V_{2,2} \)

\[
(\Delta d_s, \Delta d_u) = (1,3) \\
(n, m) = \{ \ldots \}
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\[
V \sim \rho_{30} \frac{2}{u^3s} + \rho_{32} \frac{1}{us} + \rho_{34} \frac{2u}{s} + \rho_{36} \frac{2u^3}{s} + \rho_{52} \frac{2s}{u^3} + \rho_{54} \frac{2s}{u} + \rho_{56} \frac{2}{us} + \rho_{58} \frac{2u^3s}{h_2^2} + A_{44} - A_{\text{loc}}
\]

Encoded in a partially ordered chain!

\[
\rho_{30} \rightarrow \rho_{32} \rightarrow \rho_{34} \rightarrow \rho_{36} \sim f_0^2 \text{ Constant!}
\]

\[
\rho_{52} \rightarrow \rho_{54} \rightarrow \rho_{56} \sim h_2^2 \text{ Constant!}
\]

Help us classify which terms can dominate and which are the relevant fluxes.
Example: Type IIB Weak Coupling Large Complex Structure

In Mixed Hodge Theory language: $\Pi_{0,1} \rightarrow V_{2,2}$

\[
V \sim \rho_{30} \frac{2}{u^3 s} + \rho_{32} \frac{1}{us} + \rho_{34} \frac{2}{u^3} + \rho_{36} \frac{2}{s} + \rho_{52} \frac{2}{u^3} + \rho_{54} \frac{2}{u} + \rho_{56} \frac{2}{us} + A_{44} \rightarrow A_{loc}
\]

Encoded in a partially ordered chain!

\[
\rho_{30} \rightarrow \rho_{32} \rightarrow \rho_{34} \rightarrow \rho_{36}
\]

\[
\rho_{52} \rightarrow \rho_{54}
\]

Gradient flow with $s, u \rightarrow \infty$ not possible if $V \rightarrow \infty$ \[\Rightarrow\] Set $h_3 = h_2 = 0$
Example: Type IIB Weak Coupling Large Complex Structure

In Mixed Hodge Theory language: $II_{0,1} \rightarrow V_{2,2}$

\[
\begin{align*}
(\Delta d_s, \Delta d_u) &= (1,3) \\
(n, m) &= \{ \ldots \}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Potential</th>
<th>$\beta$</th>
<th>$\gamma_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{34} \frac{u^3}{s} + A_{36} \frac{u^2}{s} + A_{52} \frac{s}{u^3} + A_{54} \frac{s}{u} + A_{44} - A_{loc}$</td>
<td>(-2,-1)</td>
<td>-</td>
</tr>
<tr>
<td>$\frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{34} \frac{u^3}{s} + A_{36} \frac{u^2}{s} + A_{52} \frac{s}{u^3} + A_{54} \frac{s}{u} + A_{44} - A_{loc}$</td>
<td>(3,1) ∗</td>
<td>$2\sqrt{\frac{2}{3}}$</td>
</tr>
<tr>
<td>$\frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{34} \frac{u^3}{s} + A_{52} \frac{s}{u^3}$</td>
<td>(1,1) ∗</td>
<td>$\sqrt{2}$</td>
</tr>
<tr>
<td>$\frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{34} \frac{u^3}{s} + A_{54} \frac{s}{u}$</td>
<td>(1,0)</td>
<td>$\sqrt{2}$</td>
</tr>
<tr>
<td>$\frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{52} \frac{s}{u^3}$</td>
<td>(2,1)</td>
<td>$\sqrt{\frac{2}{7}}$</td>
</tr>
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<td>$\frac{A_{30}}{su^3} + \frac{A_{32}}{us} + A_{34} \frac{u}{s}$</td>
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<tr>
<td>$\frac{A_{30}}{su^3}$</td>
<td>(-1,1)</td>
<td>-</td>
</tr>
</tbody>
</table>

Check Table 2 of the paper!
PART III

Results and Discussion
Asymptotic Accelerated Expansion in String Theory?

We find two potential examples of asymptotic accelerated expansion!

- Type IIB at weak coupling and large complex structure:
  \[ V \sim f^2 \frac{u}{s} + h_0^2 \frac{s}{u^3} + \ldots \quad \Rightarrow \quad \gamma = \sqrt{\frac{2}{7}} < c_{TCC} \]

- A not weakly-coupled F-theory limit:
  \[ V \sim A^{42} \frac{1}{u^2} + A^{24} \frac{u^2}{s^2} + \ldots \quad \Rightarrow \quad \gamma = \frac{2}{\sqrt{5}} < c_{\text{strong}} \]

Caveat: Have to stabilize Kähler moduli, otherwise they contribute to $\gamma$

- Required for phenomenologically viable quintessence model (5th forces)
- Need to use quantum corrections! And probably metastable!
- Caveat avoided in Type IIA... but we found no potential example there
Asymptotic Accelerated Expansion in String Theory?

**Bonus track:** Recent *exciting results!*

[Cremonini, Gonzalo, Rajaguru, Tang, Wrase ‘23]
No asymptotic accelerated expansion in a model without Kähler moduli

[Hebecker, Schreyen, Venken ‘23]
Suggest: Asymptotic accelerated expansion $\leftrightarrow$ dS in more dimensions!

$\implies$ If one is hard, the other too!

[Andriot, Tsimpis, Wrase ‘23]
Asymptotic accelerated expansion for open universes in String Theory

$\ldots$ without cosmological horizon!

$\implies$ Asymptotic accelerated expansion ok, but QG abhors cosmological horizons?
Future Directions and Conclusions

**Future directions**
- More moduli, more general limits
- More model independent approach
- Beyond asymptotic regime: Quantum corrections, Kähler stabilization

**Technically challenging, and thus exciting**!

**My conclusions/feeling**
- Accelerated expansion in String Theory will require knowledge about quantum corrections
- Accelerated expansion in quantum gravity cannot last forever
- Absence of asymptotic observables, holography? [Rudelius ‘21] [Bedroya, 23]
- QG abhors cosmological horizons? [Andriot, Tsimpis, Wrase ‘23]

**Prediction for the future or our own Universe**!

Thank you for your attention!